



## **MECHANICAL ENGINEERING MSc SEMINAR (40 min.)**

**Thursday, October 8 2026 at 14:15-14:55, Lady Davis Building, Auditorium 250**

### **Nonlinear analysis of a resonating cantilever beam**

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Cantilever resonators serve as a building block in many sensors such as atomic force microscopes, sensors of biological substances, chemicals and more. In these sensors identification of a frequency shift indicates the effect of the measurand. However, frequency shifts may also be caused by changes in the amplitude of vibrations in resonating cantilevers. Therefore, accurate modeling of mechanical nonlinearities in the free vibration response of cantilevers is of great importance. Within the framework of the linear theory of elasticity, the transverse free vibration response of a vibrating cantilever is modeled using the Euler-Bernoulli beam theory. This analytic approach neglects nonlinear effects including the coupling of transverse and axial displacements. The predictions of the linear theory are adequate for low-amplitude free vibrations, but they cannot predict the response when the amplitude of vibrations is large.

The nonlinear free vibration response of cantilever beams has been extensively investigated, and surprisingly there are conflicting conclusions in literature. Some studies suggested that the frequency of free vibrations increases with increasing amplitude of vibrations (i.e., the response is stiffening) while other studies showed that the frequency of free vibrations decreases with increasing amplitude (i.e., the response is softening).

In this seminar, two models for analyzing the free vibration response are presented. Both methods are significantly more computationally efficient relative to transient dynamic simulations of the complete 3D structure.

The first model considers the cantilever as a massless elastica with a point mass at its far edge. The free vibrations of this structure are computed by time integration of two equations of motion related to the point mass. The nonlinear nature of the elastica equation yield a free vibration frequency that depends on the amplitude of vibration.

The second model is the *DISHES* algorithm which considers the stagnation state of a synchronous motion of the cantilever as an equivalent static state of a nonlinear problem. The amplitude of deflection of the cantilever edge is postulated, and d'Alembert forces are applied throughout the structure. These body forces depend on the frequency of vibration which is unknown. The reaction force required to maintain the cantilever edge at its postulated position is computed, and the frequency of vibration which nullifies this reaction force is numerically computed.

The presentation will demonstrate how each method yields a distinct conclusion regarding the nonlinear nature of the free vibration response.