



## MECHANICAL ENGINEERING SEMINAR

**Monday, December 25, 2023 at 14:30**, D. Dan and Betty Kahn Building, Room 217

### **Multi-contact collisions are surprisingly smooth**

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Hosted by Assoc. Professor Yizhar Or

In many practical situations in robotics and biomechanics, objects collide along multiple contact points which form contacts in a state-dependent order. Examples include a human catching a basketball -- 10 contacts forming in an unknown order -- or a quadrupedal robot trotting -- forming two contacts in unknown order. Modeled as hybrid systems, these examples represent a challenge because the ensemble of trajectories spans an area where the guard functions representing the contact events intersect, meaning that over an infinitesimal change in initial conditions the system could be subject to vastly different dynamics. The hybrid systems that appear in these collision problems are not arbitrary; they have the property that the equations of motion depend only on the set of contacts created but not on the order of creation of those contacts. These are "Event Selected Systems (ESS)", and in recent work we have shown that an ESS where forces change because of position dependent guards will always have a continuously smooth state-space flow. To illustrate this we built a three legged hopper whose legs generate measurably distinct dynamics individually, yet the arbitrarily ordered triple contact leads to the same outcome up to affine approximation. In other words, even though each leg moves the robot in a different way as the contacts build up, correctly suggesting a non-differentiable flow after the first impact, the dynamics from aerial phase to after all three legs have landed are smooth -- continuous and differentiable. The implication of our results is that many multi-contact problems that may seem non-smooth and difficult to control are in fact smooth and may be amenable to conventional non-linear control approaches. The talk will introduce event selected systems, and be suitable for an audience familiar with multivariate calculus.

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