Microelectromechanical (MEMS) resonators are a building block in communication devices, hand-held devices and automotive systems. In these applications resonators are used as filters, oscillators, and sensors. Seemingly, the simplest structure of a MEMS resonator is a cantilever beam that is electrostatically actuated and sensed. In this work we investigate the dynamic response of a cantilever beam. We consider the dependence of its natural frequency on geometric proportions of the beam and on material properties. In addition, we focus on the nonlinear dependence of the natural frequency on the amplitude of its vibration. With respect to this latter effect, there is conflicting evidence in literature. Some publications suggest that the natural frequency of cantilevers increases with increasing amplitude of vibrations, while other suggest that the natural frequency decreases with increasing amplitude of vibrations. In this work we aim to settle this point.

We begin by analyzing a one-dimensional (1-D) model of the vibrating beam, and then we use insights gained from this analysis to expand our modeling by considering 2-D and then 3-D models. We perform our investigation using analytic derivation of exact solutions, and approximate numerical solutions implemented in finite differences and in finite elements codes.