DYNAMICS

ME 34010

HOMEWORK PROBLEM SETS

Mahmoud M. Safadi[1], M.B. Rubin[2]

[1]*safadi@technion.ac.il*, [2]*mbrubin@technion.ac.il*

Faculty of Mechanical Engineering

Technion – Israel Institute of Technology

Spring Semester 2016

Last revision: December 2017

Problem Set 1

Problem 1

The vertical slotted guide shown in Fig. 1.1 moves horizontally at a speed 20 [mm/s].

This causes the pin P to move in the fixed parabolic slot whose shape in given by

$$y = \frac{x^2}{b} \quad , \quad b = 160 \text{ [mm]} \quad .$$

1. Find the velocity and acceleration of P.

2. Find the velocity and acceleration of the P for the position $x = 60$ [mm].

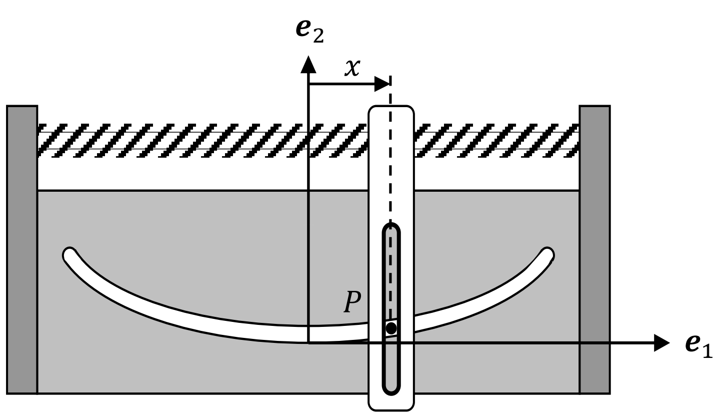

Figure 1.1

Problem 2

The absolute acceleration vector of a particle, expressed in Cartesian coordinates with basis vectors $\boldsymbol{e}_i$, is given by

$$\boldsymbol{a}(t) = (4t - 3)\boldsymbol{e}_1 + t^2\boldsymbol{e}_2 \text{ [m/s}^2\text{]} \quad .$$

The particle is initially ($t = 0$) at rest at the origin.

1. Find the velocity of the particle as a function of time.
2. Find the position of the particle as a function of time.

Problem 3

A particle passes through the points $A: (1,1,1)$ [m] and $B: (-1,4,7)$ [m] during its motion along a straight line. Let $\boldsymbol{e}_{B/A}$ denote the unit vector pointing from A to B, and $s(t)$ the distance traveled by the particle from the point A. The position vector of the particle is given by

$$\boldsymbol{x}(s) = \boldsymbol{x}_A + s\boldsymbol{e}_{B/A} = x_i(s)\boldsymbol{e}_i \text{ [m]} \quad , \quad i = \{1, 2, 3\} \quad ,$$

where the repeated index i implies a summation over i (Einstein summation convention).

1. Find the components $x_i(s)$ of $\boldsymbol{x}(s)$.
2. Let C denote the closest point to the origin along the straight line. Find the coordinates of this point.
3. Find the distance between the point C and the origin.
4. Find the distance between the points C and B.

Problem 4

A moving object is influenced by the aerodynamic drag, which is proportional to the square of the object's speed, such that the acceleration of this object is given by

$$a = -c_1 - c_2 v^2 \ [\mathrm{m/s^2}] \quad ,$$

where c_1 $[\mathrm{m/s^2}]$ and c_2 $[1/\mathrm{m}]$ are constant parameters.

The object starts its motion from the origin with speed 80 $[\mathrm{km/h}]$. Furthermore, the speeds of the object after traveling the distances of $\{200, 400\}$ $[\mathrm{m}]$ are given, respectively, by $\{60, 36\}$ $[\mathrm{km/h}]$.

Find the total distance traveled until the object stops.

Problem Set 2

Problem 1

Figure 2.1 shows a block being hauled to the surface over a curved track by a cable wound around a 750 [mm] drum, which turns at the constant clockwise speed of 120 [rpm]. The shape of the track is designed so that $y = x^2/16$, where x and y are in meters.

1. Determine the acceleration of the block as a function of x.
2. Find the magnitude of the acceleration of the block as it reaches a level of 1 [m] below the top.

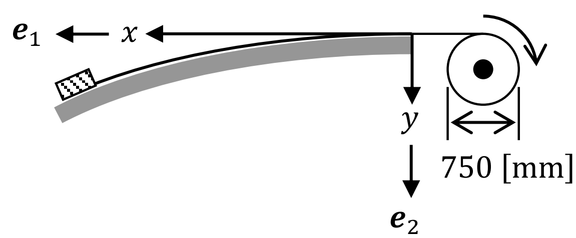

Figure 2.1

Problem 2

The pin P shown in Fig. 2.2 is constrained to move in the slotted guides A and B which move at right angles to one another. At the instant represented, A has a velocity to the right of $0.2\ [\mathrm{m/s}]$ which is decreasing at the rate of $0.75\ [\mathrm{m/s}]$ each second. At the same time, B is moving down with a velocity of $0.15\ [\mathrm{m/s}]$ which is decreasing at the rate of $0.5\ [\mathrm{m/s}]$ each second.

1. For this instant, find the radius of curvature ρ of the path followed by P.
2. Is it possible to also determine the time rate of change of ρ ?

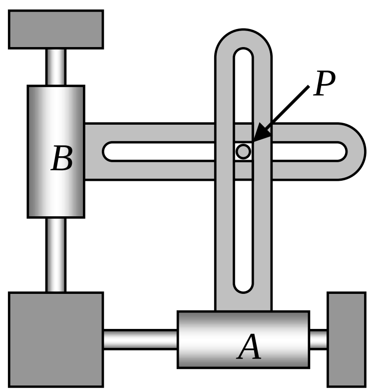

Figure 2.2

Problem 3

A particle is constrained to move along a track characterized by the function $y = 2x^{3/2}$, where x and y are in meters. The distance $s(t)$ actually traveled by the particle as it moves along the track is given by $s(t) = 2t^3$, where t denotes the time in seconds.

Initially, at the time $t = 0$, $x = 0$.

At the instant when $t = 1$ [s]:

1. Find the radius of curvature of the particle path.
2. Find the magnitude of the acceleration of the particle.

Problem 4

A particle moves in the x-y plane at constant speed b along a track characterized by the function $y = y(x)$, where x and y are in meters. Also, let s denote the actual distance traveled by the particle along the track.

1. Assuming that $dx/ds > 0$, show that

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad .$$

2. Use the chain rule of differentiation to determine the velocity of the particle as a function of x.
3. Use the chain rule of differentiation to determine the acceleration of the particle as a function of x.
4. Show that the radius of curvature at any point along the particle path is given by

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad .$$

5. Determine the unit normal vector $\boldsymbol{e}_n$ at any point along the particle path as a function of x.

Problem Set 3

Problem 1

A particle moving along a curve in space has coordinates in millimeters which vary with time t in seconds according to

$$x = 60\cos(\omega t) \quad , \quad y = 40\sin(\omega t) \quad , \quad z = 30t^2 \quad ,$$

where $\omega = 2$ [rad/s].

1. Plot the path of the particle over the time interval $0 \leq t \leq 20$ [s].

At the instant when $t = 4$ [s]:

2. Determine the unit normal and unit tangent vectors of the particle path.
3. Find the velocity of the particle.
4. Find the acceleration of the particle.
5. Find the radius of curvature of the particle path.

Problem 2

Figure 3.1 shows a particle moving along a track inside a vertical cylinder of radius $2\ [\mathrm{m}]$. At the instant represented, the particle passes through the point A with an acceleration of $10\ [\mathrm{m/s^2}]$ at an angle of 30° with respect to the horizontal plane, and it increases its speed along the track at the rate of $8\ [\mathrm{m/s}]$ each second.

For this instant:

1. Determine the velocity of the particle in terms of cylindrical-polar coordinates.
2. Find the angular speed $\dot{\theta}$ of the particle.
3. Find the angular acceleration $\ddot{\theta}$ of the particle.
4. Find the vertical component of the acceleration of the particle.

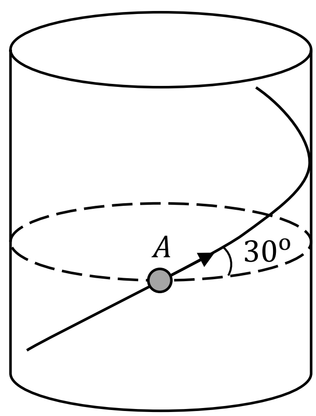

Figure 3.1

Problem 3

The cam shown in Fig. 3.2 is designed so that the center of the roller A which follows the contour moves on a limaçon defined by $r = b - c\cos(\beta)$, where $b > c$ and β is the angle between the line OB fixed to the limaçon and the slotted arm. The base vectors $\{\boldsymbol{e}_r, \boldsymbol{e}_\theta\}$ of the polar coordinate system are fixed to the slotted bar. Moreover, take $b = 100$ [mm] and $c = 75$ [mm].

At the instant when $\beta = 30^{\mathrm{o}}$:

1. Determine the total acceleration of the roller A if the slotted arm revolves with a constant counterclockwise angular speed of 40 [rpm] while the limaçon stays fixed.
2. Determine the total acceleration of the roller A if the slotted arm stays fixed while the limaçon revolves with a constant clockwise angular speed of 30 [rpm].
3. Determine the total acceleration of the roller A if the slotted arm revolves with a constant counterclockwise angular speed of 40 [rpm] while the limaçon revolves with a constant clockwise angular speed of 30 [rpm].

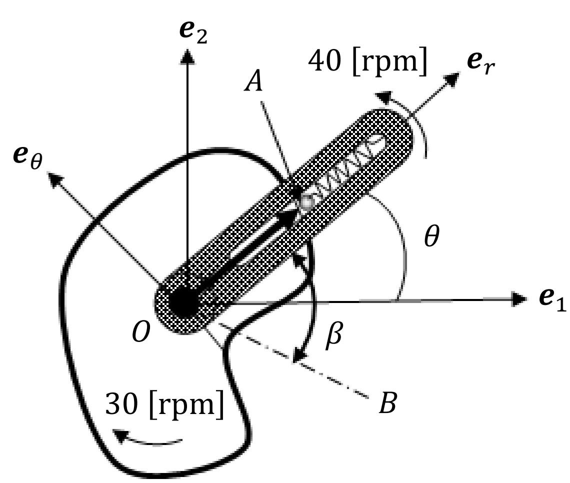

Figure 3.2

Problem 4

The hollow tube shown in Fig. 3.3 is inclined at an angle α to the vertical axis and it rotates along a circular path of radius R with a constant angular speed about the vertical axis. A particle P moves inside the tube under the control of an inextensible string which is held fixed at the point D. Moreover, the coordinate system $\boldsymbol{e}_i'$ is fixed to the tube, the distance traveled by the particle as it moves along the tube from the fixed point B is denoted by s, and the angle between the radial lines OC and OD is denoted by ϕ.

Initially, at the time $t = 0$, $\phi = 0$ and $s = 0$.

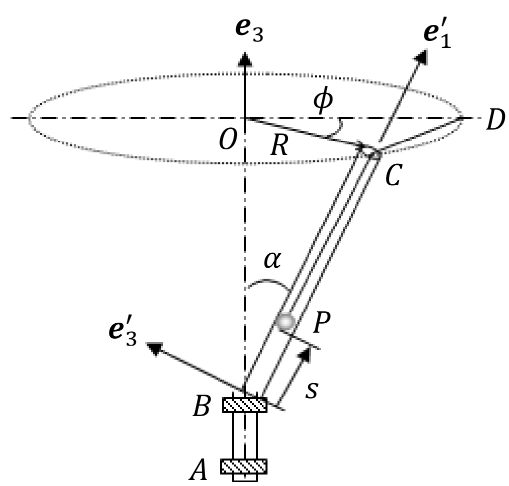

Figure 3.3

1. Determine the velocity of the particle P.
2. Determine the acceleration of the particle P.
3. Determine the velocity of the particle P along the tube.
4. Determine the acceleration of the particle P along the tube.

Problem Set 4

Problem 1

The two ends C and D of the bar CD shown in Fig. 4.1 are confined to move in the rotating slots of the right-angled frame ABF, which is hinged at B to a car that moves to the right with a constant speed v_1. The angular speed of the frame about B is $\dot{\theta}$ and is constant for the interval of motion concerned. Moreover, the whole system is accelerated upward with a constant acceleration a_0.

Initially, at the time $t = 0$, $\gamma = \theta = 0^{\text{o}}$ and the acceleration of the system is zero.

1. Determine the velocity of the midpoint E of the bar CD.
2. Determine the velocity of E relative C.
3. Determine the acceleration of E.

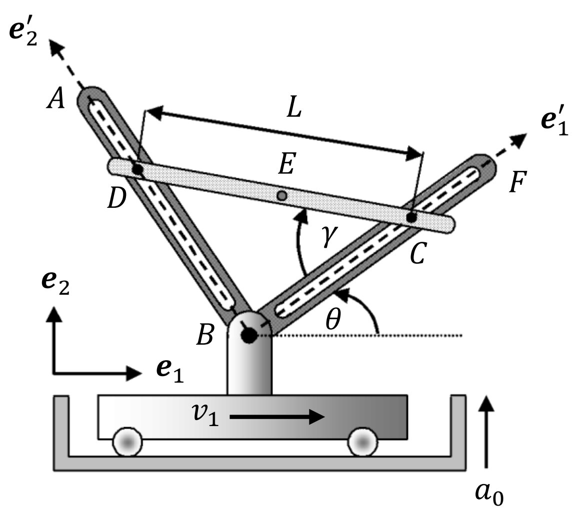

Figure 4.1

Problem 2

A car at latitude λ on the rotating earth drives straight north with a constant speed v, as shown in Fig. 4.2. The coordinate system $\boldsymbol{e}_i''$ is fixed to the earth which rotates about its axis $\boldsymbol{e}_2''$ once every 24 hours, and the coordinate system $\boldsymbol{e}_i'$ traces the motion of the car on the surface of the earth.

Determine the acceleration of the car.

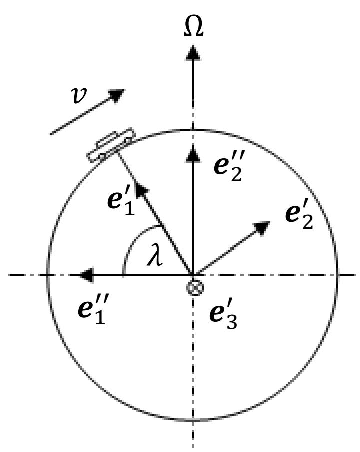

Figure 4.2

Problem 3

Consider the assembly shown in Fig. 4.3. The motor turns the disk at the constant speed $\dot{\phi}$. The motor is also swiveling about the horizontal axis that passes through the point B at the constant speed $\dot{\theta}$. Simultaneously, the assembly is rotating about the vertical axis $\boldsymbol{e}_2''$ at the constant rate $\dot{\psi}$. The system $\boldsymbol{e}_i'$ is fixed to the shaft BC, such that $\{\boldsymbol{e}_1', \boldsymbol{e}_2', \boldsymbol{e}_2''\}$ are always in the same plane.

1. Determine the angular acceleration of the disk.
2. Determine the velocity and acceleration of the center C of the disk.

Next, consider the point P which is located at a distance R from the center C of the disk.

3. Determine the velocity and acceleration of P relative to C.
4. Detertmine the velocity and acceleration of P.

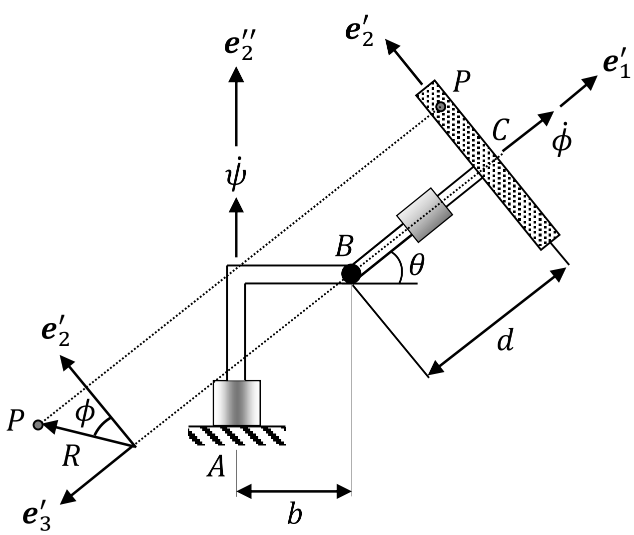

Figure 4.3

Problem Set 5

Problem 1

End A of the rigid link AB is confined to move in the negative $\boldsymbol{e}_1$ direction while end B is confined to move along the vertical axis. Determine the component $\boldsymbol{\omega}_n$ normal to AB of the angular velocity of the link as it passes the position shown in Fig. 5.1 with the speed $v_A = 0.3$ [m/s].

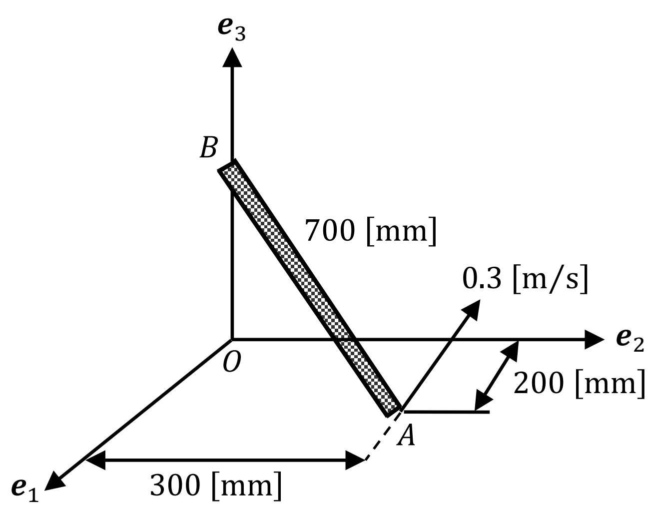

Figure 5.1

Problem 2

Determine the angular velocity of the telescoping link BC for the position shown in Fig. 5.2, where the driving links AB and CD have the angular velocities indicated.

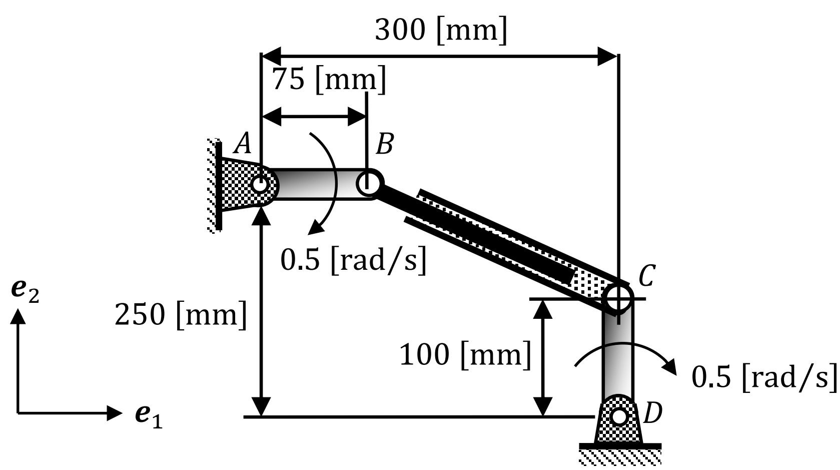

Figure 5.2

Problem 3

The slotted wheel of radius $R = 60$ [cm] shown in Fig. 5.3 rolls on the horizontal plane in a circle of radius $L = 60$ [cm]. The wheel shaft BC is pivoted at one end about an axis through the point B and is driven by the vertical shaft AB at the constant rate $\dot{\phi} = 4$ [rad/s] about the vertical axis. The slider P moves in the slot and its radial distance relative to the center of the disk is denoted by $s(t)$. The system $\{\boldsymbol{e}_i'', \boldsymbol{e}_i'\}$ are fixed to BC and the wheel, respectively, and they are always in the same plane, with θ being the angle between the axes $\boldsymbol{e}_1'$ and $\boldsymbol{e}_1''$.

1. Determine the angular velocity of the disk.
2. Determine the angular velocity of the disk for the position $\theta = 30^{\text{o}}$.
3. Determine the velocity and acceleration of the slider P.

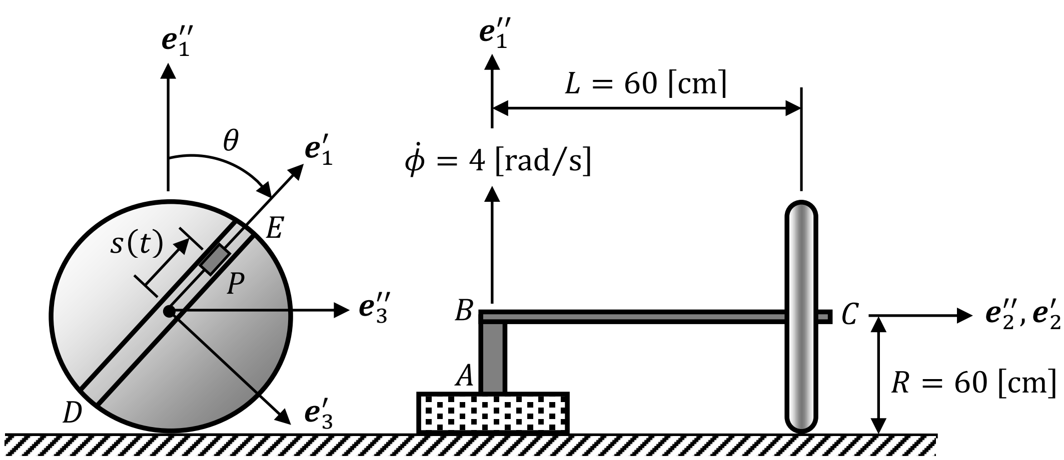

Figure 5.3

Problem 4

The hollow curved member OE shown in Fig. 5.4 rotates counterclockwise at a constant rate $\dot{\phi} = 2$ [rad/s], and the pin A causes the link BC to rotate as well. For the instant when

$$\theta = 30^{\circ} \quad , \quad \beta = 45^{\circ} \quad , \quad H = 280 \text{ [mm]} \quad , \quad L = 120 \text{ [mm]} \quad ,$$

where β is the angle between the vertical axis and the tangent to OE at A, determine the velocity of end B of the link BC.

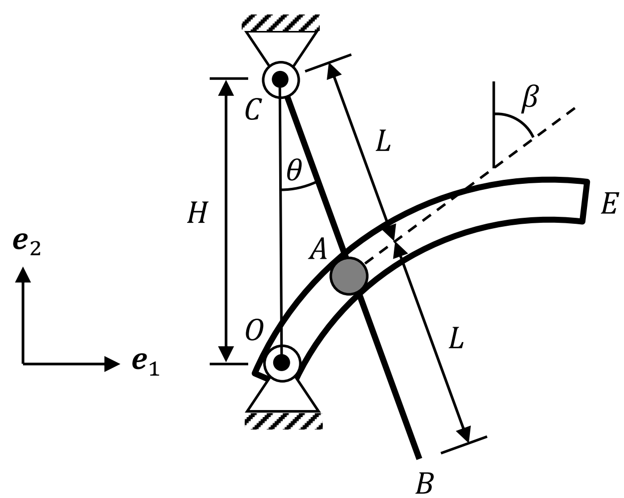

Figure 5.4

Problem Set 6

Problem 1

Fig. 6.1 shows an astronaut training facility. The drum swivels about the horizontal axis $\boldsymbol{e}_1''$ that passes through the hinge A at the rate $\dot{\beta}$. The training room is located inside the drum and it rotates about the axis $\boldsymbol{e}_1'$ at the rate $\dot{\psi}$. Simultaneously, the training facility rotates about the vertical axis $\boldsymbol{e}_2''$ at the rate Ω . At the instant when

$$\beta = 0 \quad , \quad \dot{\beta} = 0.9 \text{ [rad/s]} \quad , \quad \Omega = 0.2 \text{ [rad/s]} \quad , \quad \dot{\psi} = 0.9 \text{ [rad/s]} \quad ,$$

determine the angular velocity and acceleration of the training room.

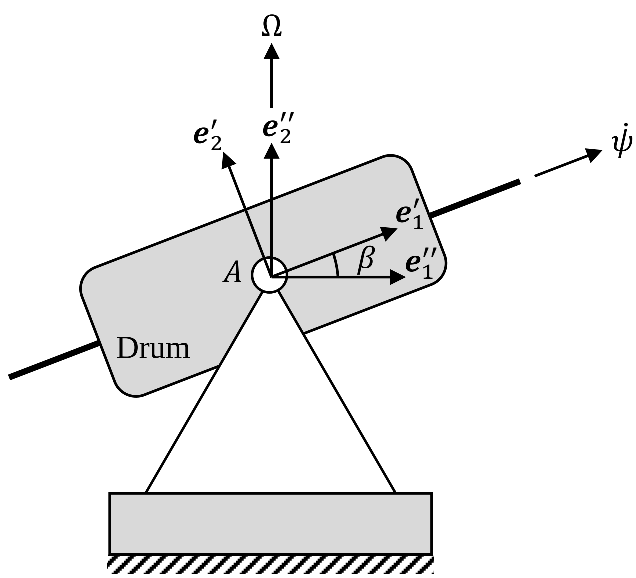

Figure 6.1

Problem 2

The 20 [kg] block A is placed on top of the 100 [kg] block B, as shown in Fig. 6.2. The block A is being pulled horizontally by a rope with a pull magnitude of P. If the coefficient of static and kinetic friction between the two blocks are both essentially the same value of 0.5, and the horizontal plane is frictionless:

1. Plot the acceleration of each block as a function of P.
2. Determine the acceleration of each block for $P = 60$ [N] and $P = 40$ [N].

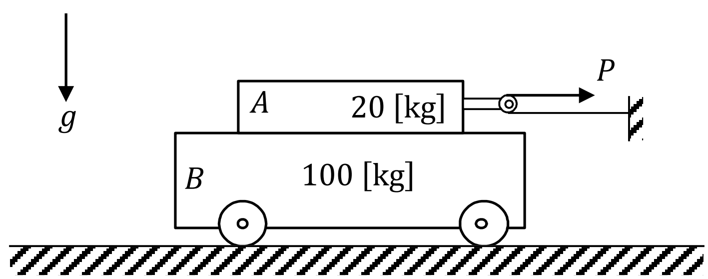

Figure 6.2

Problem 3

The sliders A and B are connected by a light rigid bar of length $l = 0.5$ [m] and move in the slots shown in Fig. 6.3. The slider A is being pulled horizontally by a constant force of magnitude $P = 40$ $[N]$. For the position where $x_A = 0.4$ [m], the velocity of A is given by $v_A = 0.9$ $[\mathrm{m}/s]$ to the right. At this instant:

1. Determine the acceleration of each slider.
2. Determine the force in the bar.

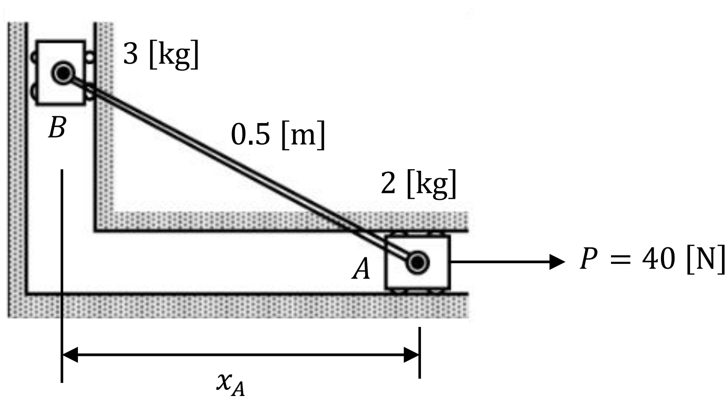

Figure 6.3

Problem 4

The small ball of mass m, shown in Fig. 6.4, is attached to a light bar of length L which swivels about the horizontal axis through B at the constant rate $\dot{\beta}$. Simultaneously, the vertical bar rotates about the vertical axis with a constant angular speed $\dot{\phi}$.

1. Determine the acceleration of the ball.
2. Determine the tension T in the bar.
3. Determine the shear force exerted on the bar by the ball.

Express your answers in terms of $\{L, \beta, \dot{\beta}, \dot{\phi}, g\}$.

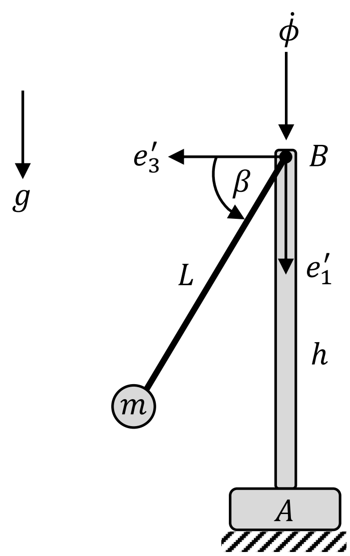

Figure 6.4

Problem Set 7

Problem 1

The two springs of stiffness 800 [N/m] and unstretched length of 0.3 [m] are attached to the collar of mass 10 [kg], which slides with negligible friction on the fixed vertical shaft under the action of gravity, as shown in Fig. 7.1. The collar is released from rest at the top position.

1. Determine the distance traveled by the collar along the vertical shaft.
2. Determine the velocity of the collar as it covers half of that distance.

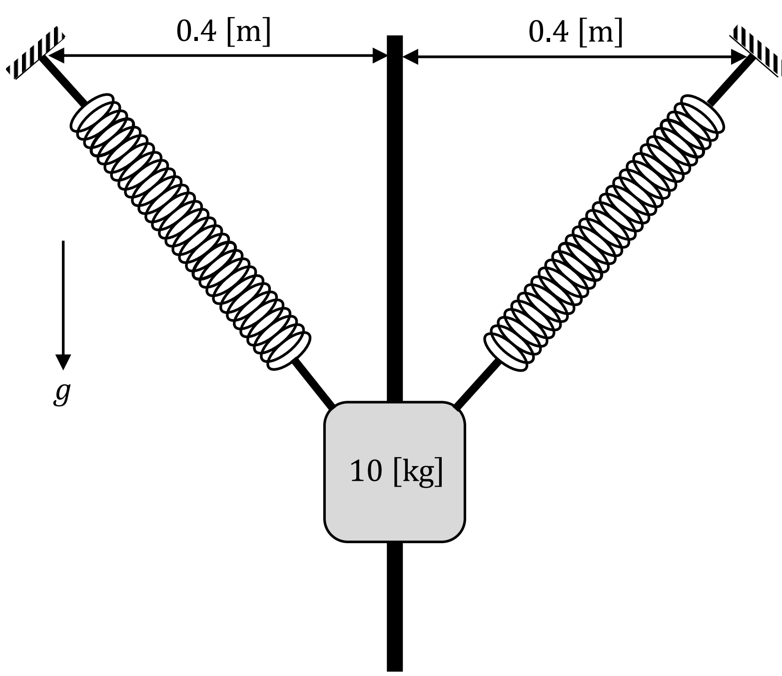

Figure 7.1

Problem 2

The 10 [kg] bead A is released from rest in the position shown in Fig. 7.2 and slides freely up the fixed circular rod AB of radius $a = 2.4$ [m] under the action of gravity and a constant force $P = 250$ [N]. Then, the bead slides on the rough horizontal rod BC with a kinetic friction of 0.5 under the action of gravity alone. Later, the bead sticks to a spring of stiffness k at the left end C of the rod BC.

1. Determine the work done by the force P on the bead from A to B.
2. Determine the velocity of the bead as it passes through the point B.
3. Determine the work done by friction on the bead from B to C.
4. Determine the value of the spring's stiffness k when it is maximally compressed by 10 [cm].

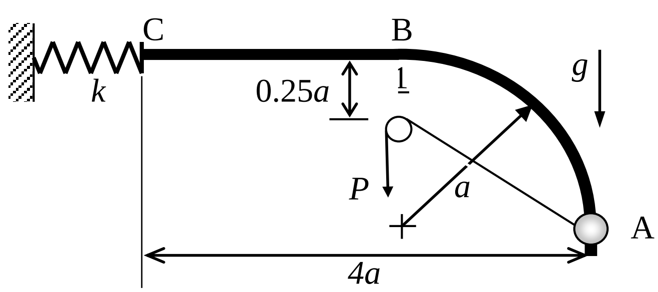

Figure 7.2

Problem 3

The small ball of mass m is attached to an inextensible rope of length L, as shown in Fig. 7.3. Initially, at the time $t = 0$, $\theta(0) = \theta_0$, $\dot{\theta}(0) = 0$, and the particle is given a velocity of $\boldsymbol{v}(0) = -v_0\boldsymbol{e}_3'$. Just afterwards, the rope begins swiveling about the horizontal axis through B at the rate $\dot{\theta}$, and the vertical bar begins rotating about the vertical axis at the rate $\dot{\phi}$. The system $\boldsymbol{e}_i''$ is fixed to the vertical bar and it lies in the plane containing the system $\boldsymbol{e}_i'$.

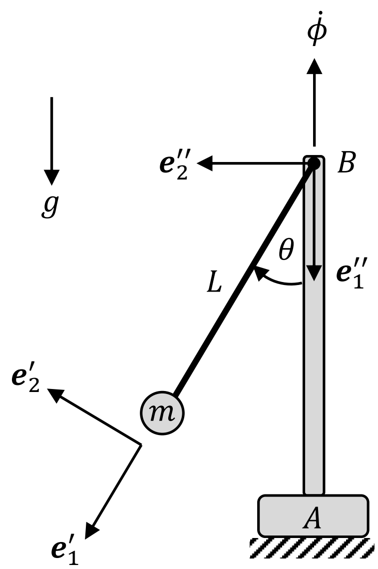

Figure 7.3

1. Is the linear momentum of the ball conserved in the $\boldsymbol{e}_3'$ direction?
2. Is the angular momentum of the ball about B conserved in the $\boldsymbol{e}_3''$ direction?
3. Is the angular momentum of the ball about B conserved in the $\boldsymbol{e}_1''$ direction?
4. Does the rope do work on the ball?
5. Determine the kinetic energy of the ball.
6. Determine the values of $\dot{\theta}$ and $\dot{\phi}$ in terms of $\{L, \theta, \theta_0, v_0, g\}$.

7. Determine the absolute acceleration of the ball.

8. Determine the tension in the rope.

Problem 4

An object of mass $m = 2$ [kg] moves on the inside of a smooth conical dish of radius $R = 3\ [m]$ and edge length of $Y = 5$ [m] while being attached to a vertical spring of stiffness $k = 300$ [N/m], as shown in Fig. 7.4. At the time $t = 0, x(0) = 4$ [m], the spring is unstretched and the object is given a velocity $v_0 = 3\ [\mathrm{m}/s]$ tangent to the horizontal rim of the surface of the dish.

1. Write down the equation of motion of the object.
2. Determine the minimal distance traveled by the particle relative to the bottom end of the dish.
3. Determine the velocity of the particle at that distance.

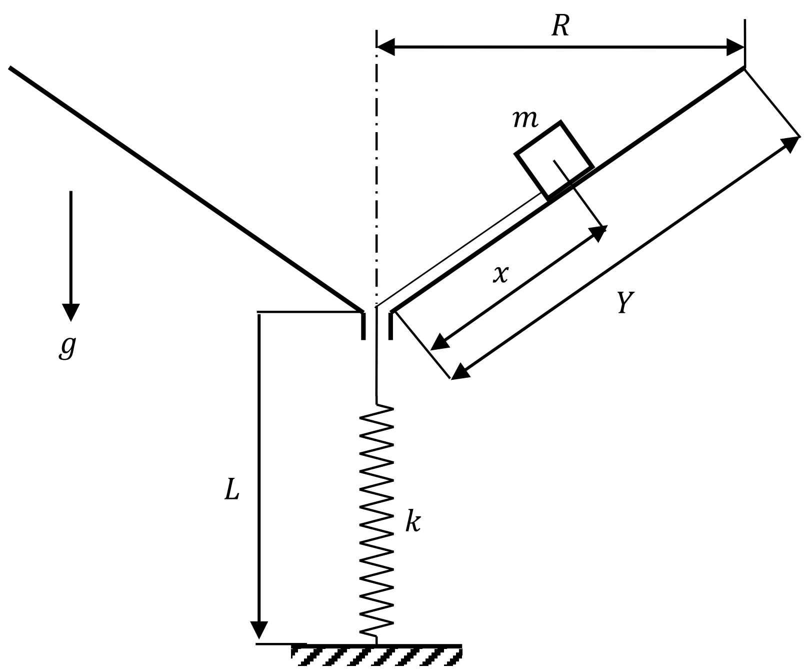

Figure 7.4

Problem Set 8

Problem 1

Figure 8.1 shows a particle of mass m, which is attached to a spring of stiffness k and free length r_0, and placed on a frictionless table. At the time $t = 0$, the spring's length is r_0 and the particle is given a velocity v_0 in the direction perpendicular to the spring.

1. Determine the equation of motion of the particle.
2. Are the linear momentum, angular momentum about the fixed point O and mechanical energy of the particle conserved?
3. Describe the motion of the particle.

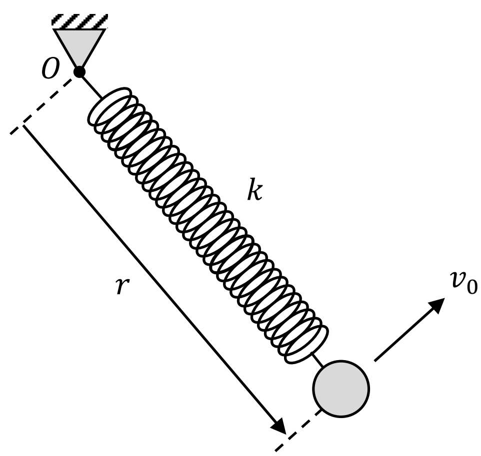

Figure 8.1

Problem 2

Figure 8.2 shows a small ball of mass m which is attached to a rigid bar AB with length L and negligible. The bar is attached at its end A to a cart of mass M, which moves horizontally along a frictionless track. Moreover, the bar rotates freely about the vertical axis passing through A. At the time $t = 0$, $\theta(0) = 0$, the velocity of the cart is v_0 and the angular velocity of the bar is ω_0.

1. Determine the velocity of the cart when $\theta = \pi$.
2. Determine the angular velocity of the bar when $\theta = \pi$.
3. Determine the maximum and minimum angular velocities of the bar.
4. Determine the maximum and minimum velocities of the cart.

Express your answers in terms of $\{m, M, R, v_0, \omega_0\}$.

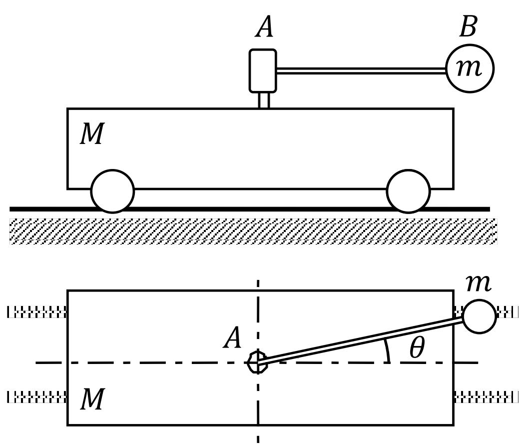

Figure 8.2

Problem 3

A tennis ball of mass m is released from rest at a height of 1600 [mm] above the ground, as shown in Fig. 8.3.

1. Determine the minimum coefficient of restitution for which the ball rises to a height of 1100 [mm] after the collision with the ground.
2. Determine the maximum energy lost in this case.

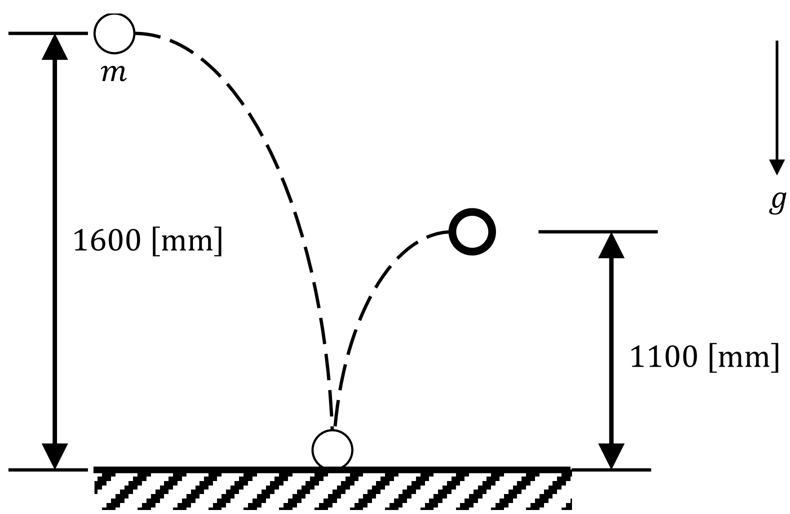

Figure 8.3

Problem 4

Figure 8.4 shows a particle of mass m_1 which is attached to the ceiling through an inextensible string of length l_1. Moreover, a particle of mass m_2 is attached to m_1 through an inextensible string of length l_2. At the time $t = 0$, m_2 is released from rest at a distance l_1 below the ceiling and the string l_2 is unstretched. At the instant when $\cos(\alpha) = 0.8$ and $\sin(\alpha) = 0.6$, the string l_2 becomes taut.

Determine the velocities of the particles just after impact, when the string l_2 becomes taut.

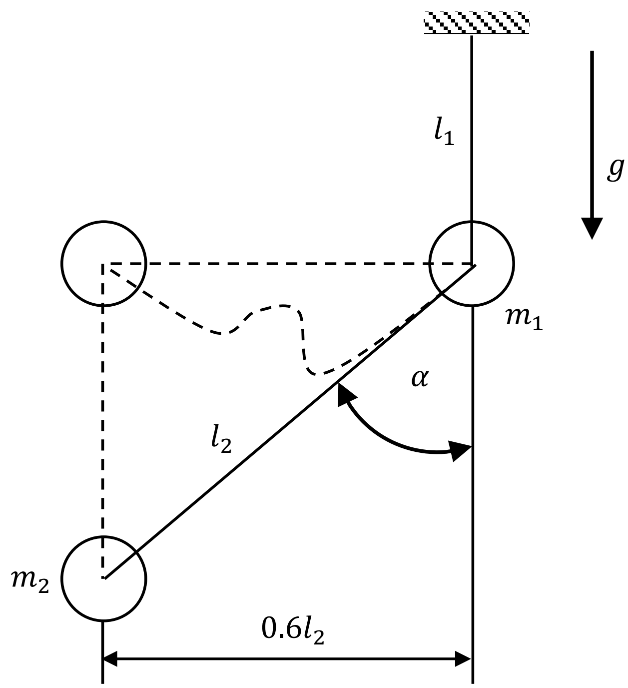

Figure 8.4

Problem Set 9

Problem 1

Fig. 9.1 shows two particles of masses $m_1 = m$ and $m_2 = 2m$, connected by a spring of stiffness k and free length L, which are initially at rest. Then, a particle of mass $m_3 = 3m$, traveling with speed v in a direction perpendicular to the spring, strikes m_1. The coefficient of restitution at impact is given by e.

1. Determine the velocity of each mass just after impact.
2. Determine the angular velocity of the line connecting m_1 and m_2 as a function of the distance $x(t)$ between these particles.
3. Determine the differential equation associated with $x(t)$.

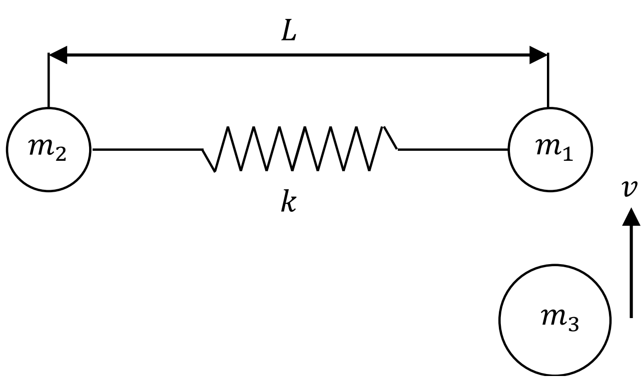

Figure 9.1

Problem 2

The upper end B of the bar AB, having a length of L and mass of m, is connected to the fixed point C by an inextensible rope, as shown in Fig. 9.2. At the time $t = 0$, the rope is cut, with $\theta(0) = \theta_0$ and $\dot{\theta}(0) = 0$.

1. Determine the angular speed $\dot{\theta}$ of the bar as a function of θ.
2. Determine the reaction forces at A.

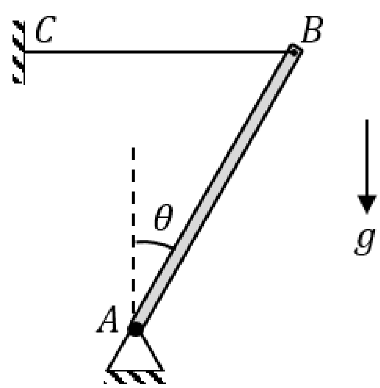

Figure 9.2

Problem 3

Fig. 9.3 shows a cylinder of mass m and radius R which is being pulled to the right by a constant horizontal force P at its center C. Initially, at the time $t = 0$, $x(0) = 0$, $\dot{x}(0) = -\omega_0 R$, $\theta(0) = 0$ and $\dot{\theta}(0) = \omega_0 > 0$.

Determine $x(t)$ and $\theta(t)$ and the magnitude of the friction force between the cylinder and the ground for the following cases:

1. $P = 0$.
2. $P = 2\mu m g$.
3. $P = 4\mu m g$.

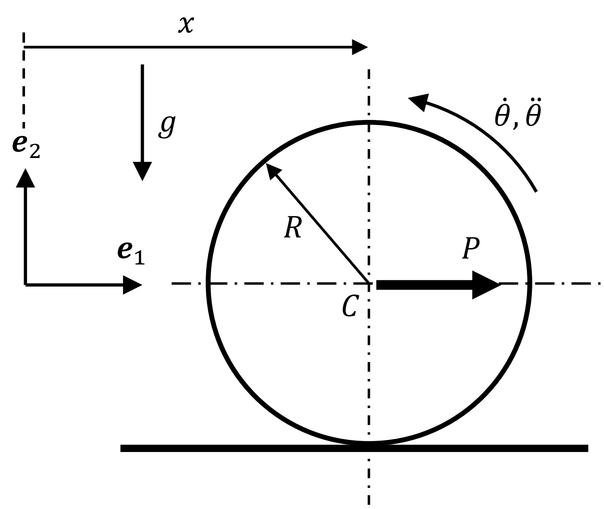

Figure 9.3

Problem 4

A bowling ball of mass m and radius R is thrown onto the ground with a velocity v_0 that is essentially horizontal. The friction coefficient between the ball and the ground is μ. Initially, at the time $t = 0$, $\theta(0) = 0$ and $\dot{\theta}(0) = 0$.

Determine the distance traveled by the ball before it starts rolling without slipping on the ground.

Problem Set 10

Problem 1

Consider the assembly shown in Fig. 10.1. The hanging block of mass m_1 is attached to the cylinder of center B, mass m_2 and radius r_2 by an inextensible cord, wrapped at a radius r_1 and passes over a drum of center A, mass m_3 and radius R.

It is assumed that the cord does not slip on the drum and the cylinder. Moreover, the coefficient of friction between the cylinder and the ground is μ.

1. Assuming that the cylinder rolls without slipping along the ground, determine the acceleration of the block.
2. Assuming that

   $\{\mu = 0.3, m_1 = m, m_2 = m/2\,, r_1 = r, r_2 = 2r, \bar{I}_2 = 6mr^2, \bar{I}_3 = 3mr^2, R = r_1 + r_2\}$,

   show that the cylinder slips along the ground. Also, determine the acceleration of the block.

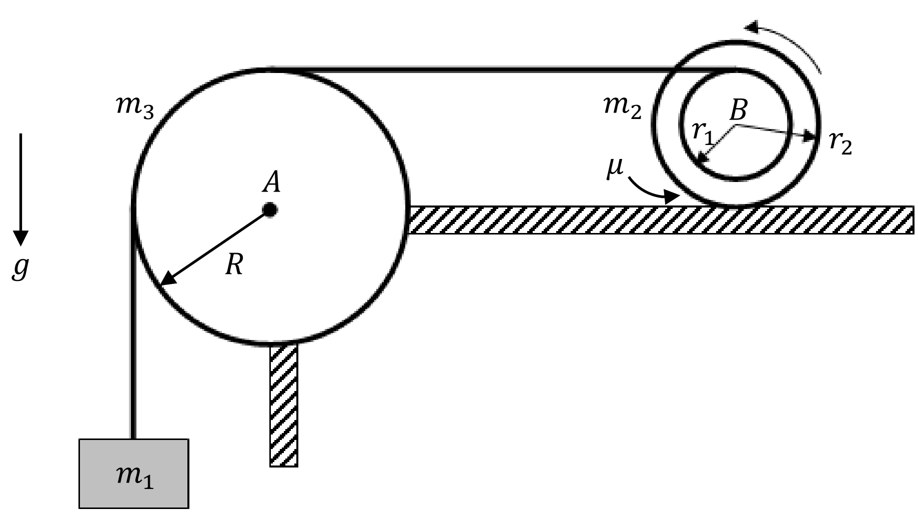

Figure 10.1

Problem 2

Figure 10.2 shows a cylinder of center C, mass m_1 and radius R which is placed on a stationary box of mass m_2. The coefficient of friction between the cylinder and the box is μ. At the time $t = 0$, a block of mass m_1, moving freely with a leftward velocity of v, strikes the box and sticks to it.

1. Assuming that $\mu = 0$, determine the velocities of the box and the center C of the cylinder just after impact.
2. Assuming that $\mu > 0$ and the cylinder slips on the box during impact, determine the velocities of the box and the center C of the cylinder just after impact.
3. Using your answers in part 2, determine the time it takes for the cylinder to begin rolling without slipping on the box.
4. Assuming that $\mu \to \infty$, determine the velocities of the box and the center C of the cylinder just after impact.

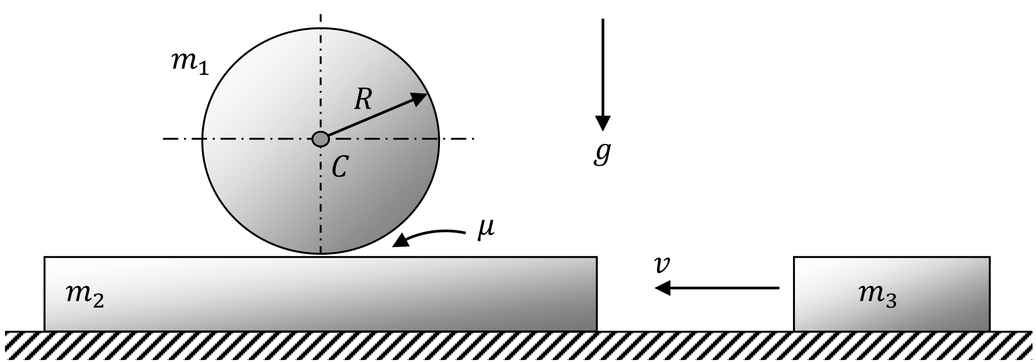

Figure 10.2

Problem 3

Figure 10.3 shows a disk of mass m and radius b, which is attached to a frame by an inextensible cord of length $3b$ passing through its center C. The frame rotates with a constant angular acceleration $\ddot{\theta} = p$. The coefficient of friction between the disk and the frame at the point of contact B is μ. Initially, at the time $t = 0$, $\theta(0) = 0$ and both the disk and the frame are at rest. The maximum tension in the cord is given by T_{cr}. Also, gravity is neglected.

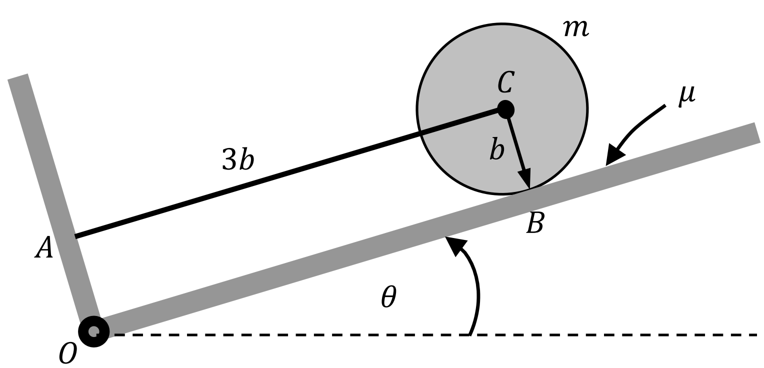

Figure 10.3

1. Assuming that $\mu = 0$, determine:

1.1. the angular velocity of the disk.

1.2. the tension in the cord.

1.3. the angular velocity of the frame when the disk is on the verge of bouncing off.

2. Determine the critical value of μ, denoted by μ_{cr}, for which the disk slips on the frame at the onset of motion.

3. If $\mu > \mu_{cr}$, determine the angular velocity of the frame when the disk is on the verge of slipping.

4. Assuming that the disk does not slip on the frame, determine:

4.1. the kinetic energy of the disk.

4.2. the angular momentum of the disk about the fixed point O.

4.3. the minimum value of the angular acceleration of the disk for which the cord tears at the onset of motion.

Problem 4

Consider the assembly shown in Fig. 10.4. The hanging block of mass m_2 is attached to the cylinder of center C, mass m_1 and radius R by an inextensible cord, wrapped around the cylinder and passes over a massless pulley. The coefficient of friction between the cylinder and the ground is μ. Moreover, the system is released from rest with the cylinder being at a distance $4R$ relative to the fixed vertical wall. The coefficient of restitution between the vertical wall and the cylinder is given by $e = 1/2$.

1. Assuming that the cylinder rolls without slipping along the ground, determine:

1.1. the acceleration of its center C at the onset of motion.

1.2. the minimum value of μ for this to happen.

1.3. the velocity of its center C just before impact with the wall.

2. Determine the velocity of the center C of the cylinder just after impact with the wall.
3. Determine the angular velocity of the cylinder just after impact with the wall.
4. Does the cylinder slip along the ground just after impact with the wall? Explain your answer.

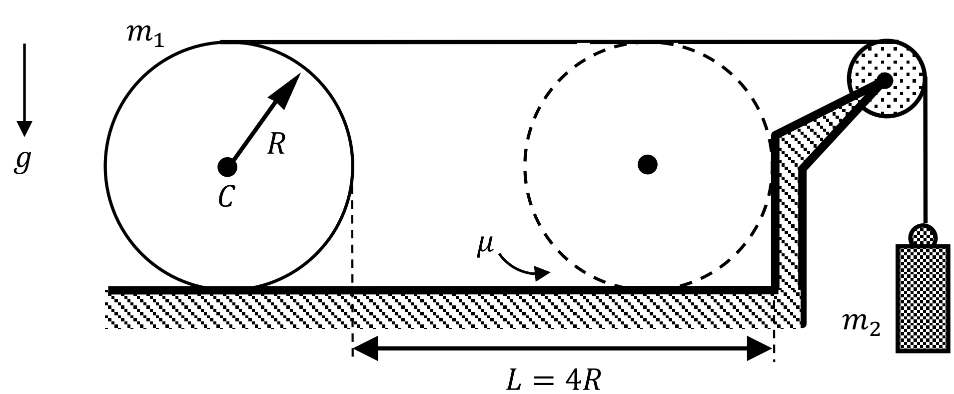

Figure 10.4

Problem Set 11

Problem 1

A uniform circular disk of mass $m = 23$ [kg] and radius $R = 0.4$ [m] rolls without slipping along a horizontal surface in such a manner that its plane is inclined with the vertical at a constant angle α and its center C moves along a circular path of radius $b = 0.6$ [m] with the speed $v = 2.54$ [m/s], as shown in Fig. 11.1.

1. Determine the value of α.
2. Determine the forces exerted on the disk by the horizontal surface.

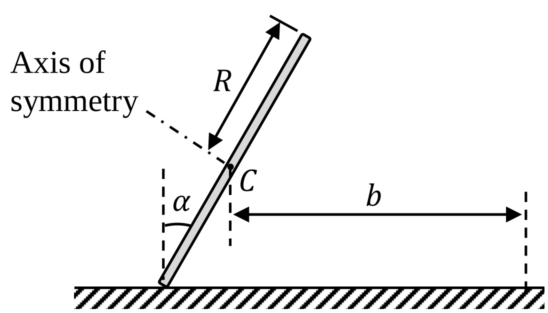

Figure 11.1

Problem 2

Consider the assembly shown in Fig. 11.2. The two disks A and B are welded at the two ends of the shaft AB of length $2L$, which coincides with the axis of symmetry of each disk. A third disk, C, is welded at the midpoint of the shaft in such a manner that its plane is inclined with the horizontal at a constant angle β. Moreover, the system $\boldsymbol{e}_i''$ is attached to the shaft and it is assumed that the torques at the bearing A and B are negligible.

1. Determine the angular momentum of the system about C.
2. Determine the normal bearing reactions acting on the shaft AB at A and B.
3. Now, the point masses m_A and m_B are attached at the rim of the disks A and B, respectively (see Fig. 2.1). Determine the values of $\{m_A, m_B\}$ and $\{\varphi_A, \varphi_B\}$ that would eliminate the bearing reactions at A and B.

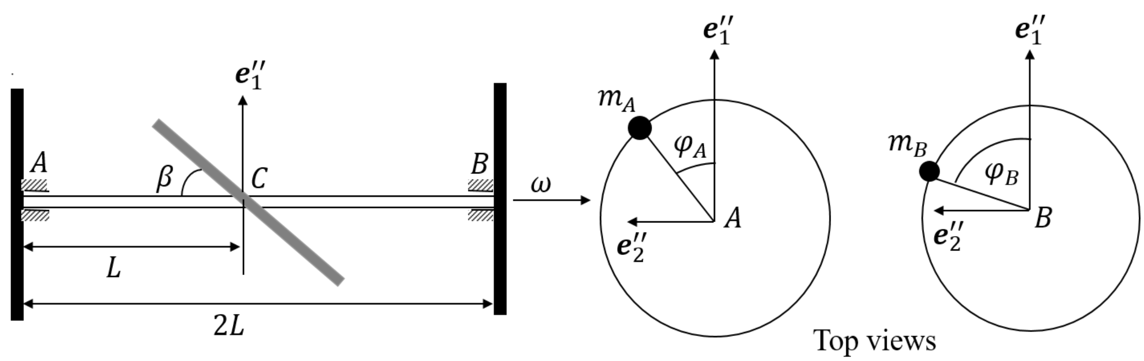

Figure 11.2

Problem 3

Fig. 11.3 shows a bar AC of mass m and length L, which is attached at one end to the center C of a disk of mass m and radius R, and the other end is placed on a stationary, frictionless circular plate at the point A. The bar coincides with the axis of symmetry of the disk. Moreover, the disk is constrained to roll without slipping along the rim of the plate in such a manner that its center C moves along a circular path with the speed v_0.

1. Determine the angular velocity and acceleration of the disk.
2. Determine the forces exerted on the disk by the plate.

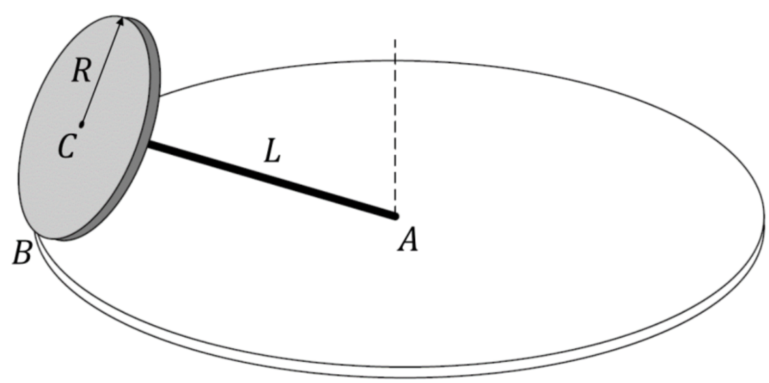

Figure 11.3