Continuous flexible structures are employed in various advanced engineering applications, such as unmanned space or aerial vehicles, deep well drilling and medical instruments. Flexibility means light weight structures. However, it may yield undesired vibrations even for slow tracking maneuvers or disturbance signals. While passive rigidization simply results in heavier structures, the alternative is active rigidization via control algorithms. This work considers finite length structures that have no resistance to bending, such as strings (equivalent to rods in tension or torsion) or membranes. They are described by the wave equation of one and two spatial dimensions, with or without domain damping. First we treat the free response of the wave equation with boundary damping, for which the literature falls short of providing closed form solutions. We obtain it in a traveling wave form, extending the classical d’Alembert solution. With a new orthogonality condition we derive the explicit modal series solution.

The main result of this research is a control method that exploits the system wave oriented properties. First it involves modeling by infinite dimensional transfer functions (TFs), which contain different fractional order expressions of the complex variable. Their time domain equivalents are Bessel functions, which we show to exhibit the wave shape evolution during motion, non-pure delays, the effect of reflection from the boundaries etc. The control algorithm involves boundary actuation and measurement and contains three building blocks. The first is a controller that eliminates the wave reflections from the system, thus suppressing its vibratory modes and actively rigidizing it. The resulting TF contains a single fractional order exponent at the numerator. This absolute vibration suppression (AVS) controller was designed in previous work for the classical wave equation with general BC, which is a particular (non-fractional) case of the systems considered here. An additional loop is then constructed, which cancels the remaining fractional order delay and places the closed loop poles in any desired locations. This is an extension of the classical dead time compensator (DTC) to the fractional order realm. Finally, a pre-compensator is designed to produce a rational tracking TF with a pure delay, independent of the systems fractional components.