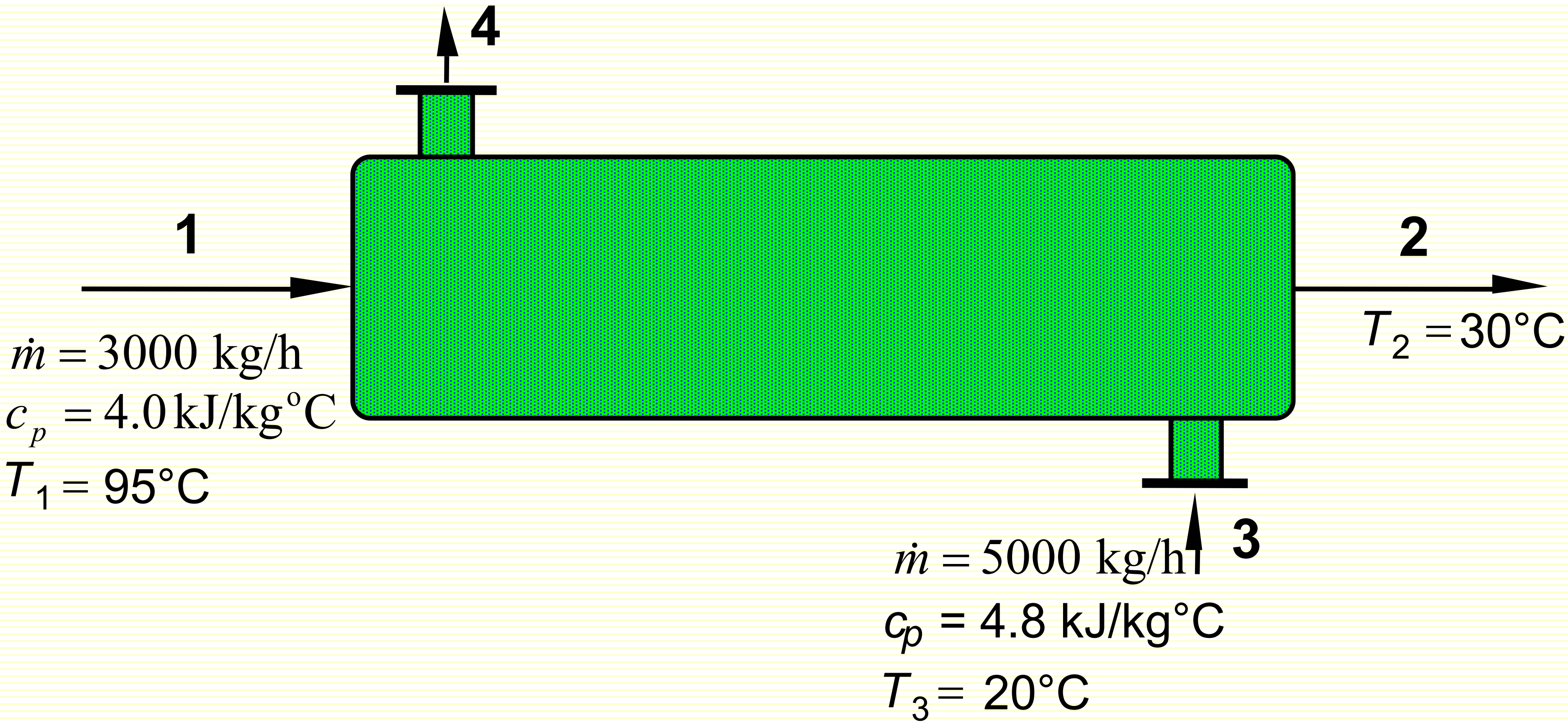
HEAT TRANSFER TRANSPARENCIES

Professor Chaim Gutfinger
Faculty of Mechanical Engineering
Technion - Israel Institute of Technology

Heat Exchanger



What is the exit temperature T_4?

Fourier Law of Heat Conduction

$$\mathbf{q} = -k \ \nabla T$$

In cartesian coordinates

$$q_x = -k \frac{\partial T}{\partial x} \qquad q_y = -k \frac{\partial T}{\partial y} \qquad q_z = -k \frac{\partial T}{\partial z}$$

in cylindrical coordinates

$$q_r = -k \frac{\partial T}{\partial r} \qquad q_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta} \qquad q_z = -k \frac{\partial T}{\partial z}$$

in spherical coordinates

$$q_r = -k \frac{\partial T}{\partial r} \qquad q_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta} \qquad q_\phi = -k \frac{1}{r \sin\theta} \cdot \frac{\partial T}{\partial \phi}$$

The Conduction Equation

Conduction equation

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot \left(k \nabla T \right) + \dot{q}$$

For *k* = const:

$$\rho c \frac{\partial T}{\partial t} = k\, \nabla^2 T + \dot{q}$$

In cartesian coordinates

$$\rho c \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{q}$$

in cylindrical coordinates

$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \frac{(r\, \partial T)}{\partial r} + \frac{1}{r} \left(\frac{\partial^2 T}{\partial \theta^2} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \dot{q}$$

in spherical coordinates

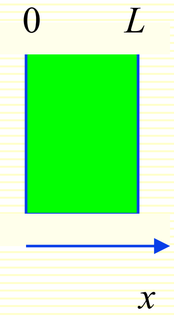
$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right] + \dot{q}$$

One-dimensional conduction

In cartesian coordinates

$$\frac{\partial^2 T}{\partial x^2} = 0 \qquad \text{b.c.} \qquad T(0) = T_1, \quad T(L) = T_2$$

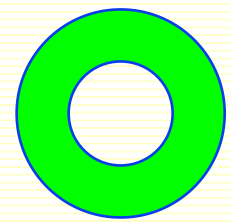
$$T = \frac{T_2 - T_1}{L} x + T_1 \qquad \dot{Q} = -kA\frac{dT}{dx} = kA\frac{T_1 - T_2}{L}$$



in cylindrical coordinates

$$\frac{\partial}{\partial r}\frac{(r\,\partial T\,)}{\partial r} = 0, \quad \text{b. c.} \quad T(r_1) = T_1, \;\; T(r_2) = T_2$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln(\mathrm{r}/\mathrm{r}_1)}{\ln(\mathrm{r}_2/\mathrm{r}_1)} \qquad \dot{Q} = -k\,2\pi rL\frac{dT}{dr} = \frac{2\pi kL}{\ln(\mathrm{r}_2/\mathrm{r}_1)}(T_1 - T_2)$$

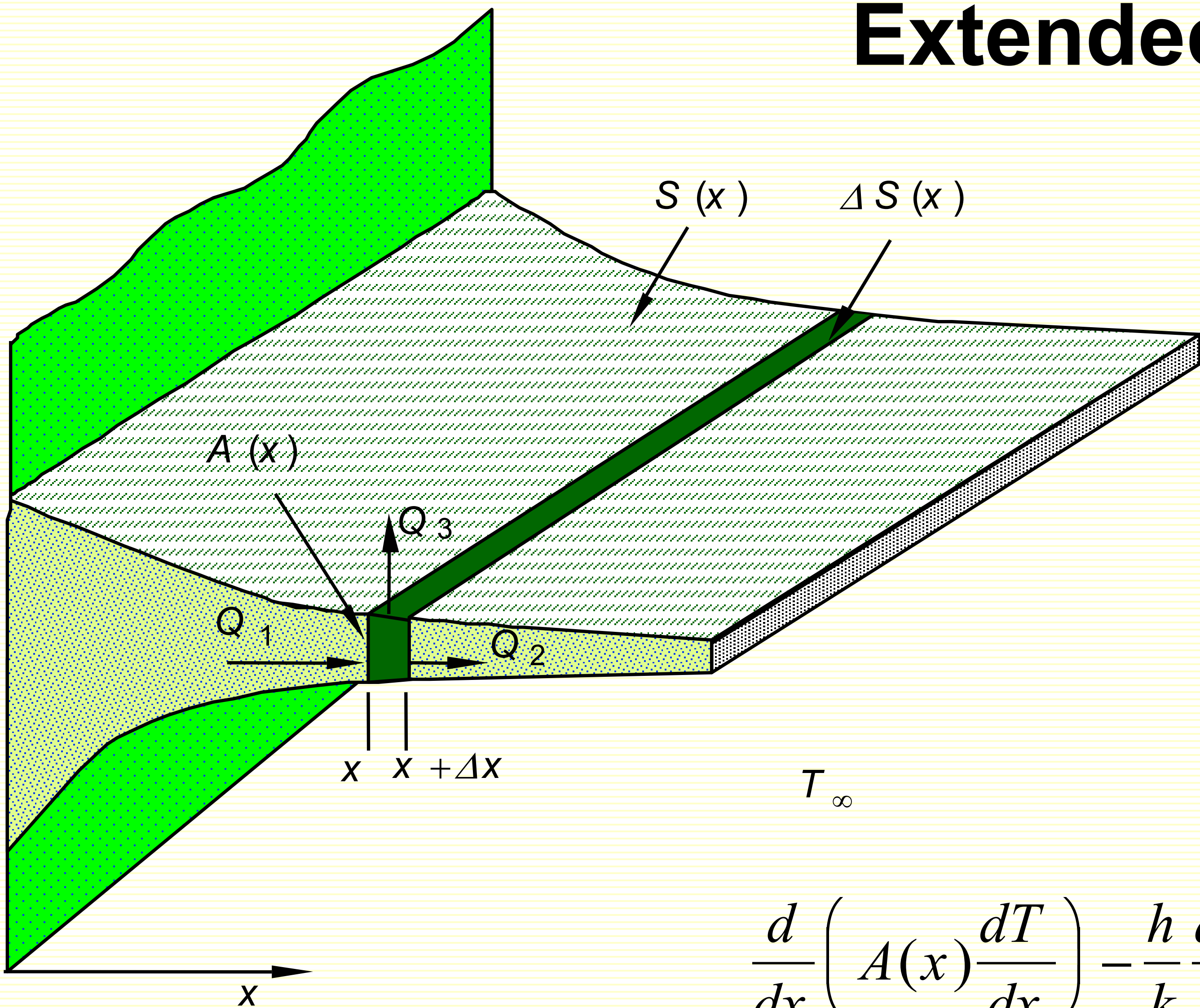


in spherical coordinates

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0, \qquad \text{b. c.} \qquad T(r_1) = T_1, \;\; T(r_2) = T_2$$

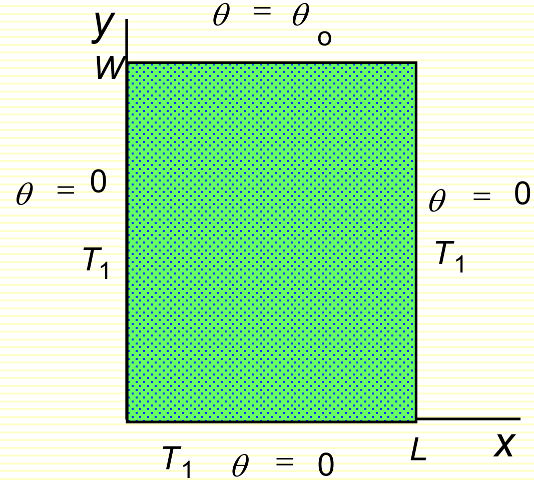
$$T = \frac{T_1 - T_2}{1/\mathrm{r}_1 - 1/\mathrm{r}_2}\frac{1}{r} + T_1 - \frac{T_1 - T_2}{1 - \mathrm{r}_1/\mathrm{r}_2}, \quad \dot{Q} = -k\,4\pi r^2\frac{dT}{dr} = \frac{4\pi k}{1/\mathrm{r}_1 - 1/\mathrm{r}_2}(T_1 - T_2)$$

Extended Surfaces



$$\frac{d}{dx}\left(A(x)\frac{dT}{dx}\right)-\frac{h}{k}\frac{dS(x)}{dx}(T-T_{\infty})=0$$

2-D steady state conduction



Rectangular plate:

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \quad ; \qquad \theta = T - T_1$$

boundary cond.

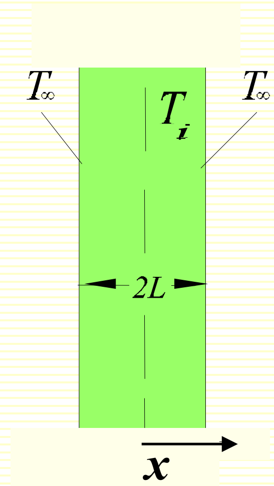
$$\theta(0, y) = 0, \qquad \theta(L, y) = 0$$

$$\theta(x, 0) = 0, \qquad \theta(x, W) = \theta_o$$

Solution by separation of variables:

$$\frac{\theta}{\theta_o} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin\left(\frac{n \pi x}{L}\right) \frac{\sinh(n \pi y / L)}{\sinh(n \pi W / L)}$$

Unsteady state conduction



Equation
$$\frac{\partial \theta^*}{\partial t^*} = \frac{\partial^2 \theta^*}{\partial x^{*2}}; \qquad \theta^* = \frac{T - T_\infty}{T_i - T_\infty},\ x^* = \frac{x}{L},\ t^* = \frac{\alpha t}{L^2} \equiv Fo$$

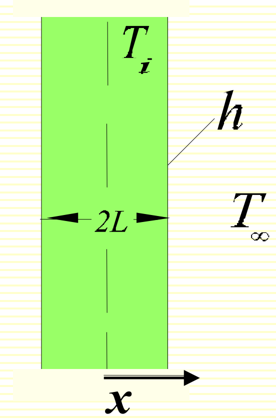
with
$$\left.\frac{\partial \theta^*}{\partial x}\right|_{x=0} = 0, \qquad \theta^*\left(1, t^*\right) = 0; \qquad \theta^*\left(x^*, 0\right) = 1$$

Solution
$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = 2\sum_{n=0}^{\infty} \frac{(-1)^n}{\lambda_n L} \cos(\lambda_n x) e^{-\lambda_n^2 \alpha t} \qquad \lambda_n = \frac{(2n+1)\pi}{2L}$$

Heat flux
$$q = -k \left.\frac{\partial T}{\partial x}\right|_{x=L} = \frac{2(T_i - T_\infty)}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\lambda_n^2 \alpha t}$$

Heat loss
$$\frac{Q}{Q_i} = \frac{2}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{\left(\frac{2n+1}{2}\right)^2} \left[1 - e^{-\lambda_n^2 \alpha t}\right]$$

Convective boundary condition



Equation $$\frac{\partial \theta^*}{\partial t^*} = \frac{\partial^2 \theta^*}{\partial x^{*2}}; \qquad \theta^* = \frac{T - T_\infty}{T_i - T_\infty},\ x^* = \frac{x}{L},\ t^* = \frac{\alpha t}{L^2}$$

Boundary cond. $$\frac{\partial \theta^*}{\partial x^*}\left(0, t^*\right) = 0, \qquad -\theta^*\left(1, t^*\right) = \left(\frac{k}{hL}\right)\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1}$$

Initial condition $$\theta^*\left(x^*, 0\right) = 1$$

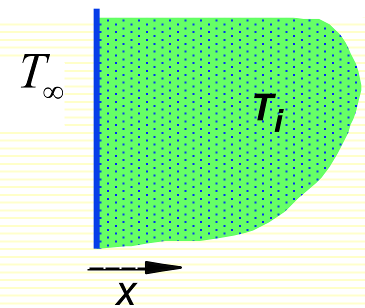
Solution $$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \frac{2}{\pi}\sum_{n=1}^{\infty} C_n \cos\left(\zeta_n x^*\right)\exp\left(-\zeta_n^2 t^*\right)$$

where $$C_n = \frac{4\sin\zeta_n}{2\zeta_n + \sin 2\zeta_n} \quad ; \quad \zeta_n = \lambda_n L \quad ; \quad \zeta_n \tan\zeta_n = Bi$$

Approximate solution

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = C_1 \cos\left(\zeta_1 x^*\right)\ \exp\left(-\zeta_1^2 t^*\right) = \frac{\theta_o}{\theta_i}\cos\left(\zeta_1 x^*\right)$$

Semi-infinite solid



Equation

$$\frac{\partial\theta}{\partial t} = \alpha \frac{\partial^2\theta}{\partial x^2}; \qquad \text{where} \qquad \theta = \frac{T - T_i}{T_\infty - T_i}$$

Boundary & initial cond.

$$\theta(0,t) = 1, \qquad \theta(\infty,t) = 0; \qquad \theta(x,0) = 0$$

Similarity solution

$$\frac{d^2\theta}{d\eta^2} + 2\eta\frac{d\theta}{d\eta} = 0 \qquad \text{where} \qquad \eta = \frac{x}{2\sqrt{\alpha t}}$$

$$\theta(0) = 1, \qquad \theta(\infty) = 0$$

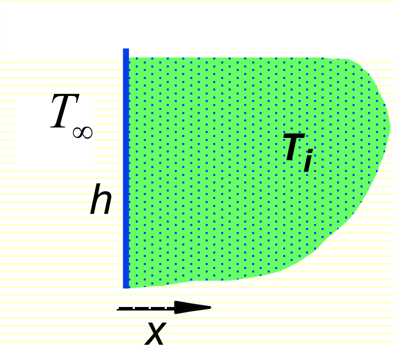
Solution

$$\theta = \operatorname{erfc}\eta \qquad \textbf{or} \qquad \frac{T - T_i}{T_\infty - T_i} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Heat flow at surface

$$\dot{Q} = -kA\left.\frac{dT}{dx}\right|_{x=0} = \frac{kA(T_\infty - T_i)}{\sqrt{\pi\alpha t}}$$

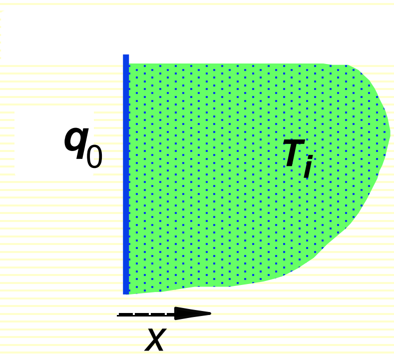
Other boundary conditions



Constant surface heat flux: $q_s = q_0$

Solution

$$T - T_i = \frac{2q_0\sqrt{\alpha t/\pi}}{k}\exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q_0 x}{k}\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



Surface convection: $-k\left.\frac{dT}{dx}\right|_{x=0} = h(T_\infty - T)$

Solution

$$\frac{T - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\right]\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

Navier–Stokes Equations

Navier–Stokes equation (incompressible)

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v}$$

In cartesian coordinates

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right),$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right),$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right).$$

Thermal energy equation

The conduction equation

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{q}$$

The energy equation adds to this convection and dissipation

$$\rho c \frac{DT}{Dt} = k \nabla^2 T + \dot{q} + \mu \Phi_v$$

where $$\mu \Phi_v = \tau_{ij} \frac{\partial v_i}{\partial x_j}$$

Two-dimensional case:

$$\rho c \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \dot{q}$$

$$+ \mu \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 \right]$$

Boundary layer on a flat plate

Momentum equation

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0$$

Integral momentum equation

$$\frac{d}{dx}\int_0^{\delta}(U-u)\,u\,dy=\nu\left.\frac{\partial u}{\partial y}\right|_{y=0}=\frac{\tau_w}{\rho}$$

Integral energy equation

$$\frac{d}{dx}\int_0^{\delta}(T_\infty-T)\,u\,dy=\alpha\left.\frac{\partial T}{\partial y}\right|_{y=0}=\frac{q_w}{\rho c}$$

Integral continuity equation

$$\upsilon=-\int_0^{y}\frac{\partial u}{\partial x}\,dy$$

Boundary layer integral solution

Boundary conditions

$$y = 0 \qquad u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0$$

$$y = \delta \qquad u = U, \quad \frac{\partial u}{\partial y} = 0$$

Velocity profile

$$u = C_1 + C_2 y + C_3 y^2 + C_4 y^3$$

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Wall shear stress

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{3}{2}\frac{\mu U}{\delta}$$

Integral momentum solution

The equation $$\frac{d}{dx}\left(\frac{39}{280}\rho U^2\ \delta\right) = \frac{3}{2}\frac{\mu U}{\delta}$$ **with b.c.** $\delta(0) = 0$

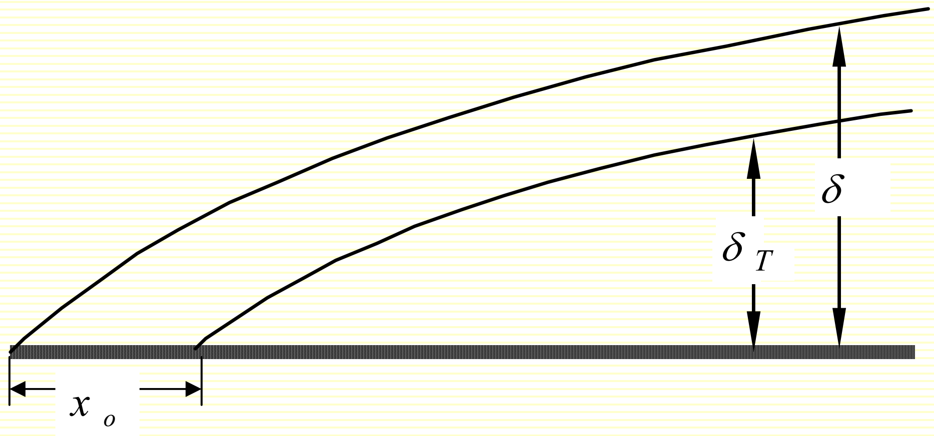
The solution $$\delta = 4.64\sqrt{\frac{\nu}{U}\ x} = \frac{4.64}{\sqrt{\mathrm{Re}_x}}\ x\ .$$

Shear stress $$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{3}{2}\mu\frac{U}{\delta} = 0.323\mu U\sqrt{\frac{U}{\nu x}} = 0.323\frac{\rho U^2}{\sqrt{\mathrm{Re}_x}}$$

Drag coefficient $$\frac{C_f}{2} = \frac{\tau_w}{\rho U^2} = 0.323\mathrm{Re}_x^{-\frac{1}{2}}\ .$$

Average drag coefficient $$\overline{C_f} = 2C_f\Big|_{x=L} = 1.292\ \mathrm{Re}_x^{-\frac{1}{2}}\ .$$

Integral thermal boundary layer



The equation

$$\frac{d}{dx}\int_0^h (T_\infty - T)u\,dy = \alpha \left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{q_w}{\rho c}$$

Temperature profile

$$\theta = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_T}\right) - \frac{1}{2}\left(\frac{y}{\delta_T}\right)^3$$

Solution

$$\frac{\delta_T}{\delta} = \frac{\left[1 - \left(x_o / x\right)^{3/4}\right]^{1/3}}{1.026\ \mathrm{Pr}^{1/3}}$$

The heat transfer coefficient

$$q_w = -k \left.\frac{\partial T}{\partial y}\right|_{y=o} = h\left(T_w - T_\infty\right)$$

$$h = \frac{-k \left.\frac{\partial T}{\partial y}\right|_{y=o}}{\left(T_w - T_\infty\right)} = k \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = \frac{3}{2}\frac{k}{\delta_T} = \frac{3}{2} \times 1.026\ \mathrm{Pr}^{1/3} \frac{\sqrt{\mathrm{Re}_x}}{4.64x}$$

$$h = 0.332k\left(\frac{U}{\nu x}\right)^{1/2} \mathrm{Pr}^{1/3}$$

$$\bar{h} = \frac{1}{L}\int_0^L h dx = 2\left.h\right|_{x=L}$$

$$\mathrm{Nu}_x = \frac{hx}{k} = 0.332\ \mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$$

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}\,L}{k} = 2\ \left.\mathrm{Nu}\right|_{x=L}$$

Turbulent boundary layers

Turbulent momentum transfer $$\frac{\tau_w}{\rho} = (\nu + \varepsilon_M)\frac{\partial u}{\partial y}$$

Turbulent heat transfer $$\frac{q_w}{\rho c} = (\alpha + \varepsilon_H)\frac{\partial T}{\partial y}$$

Integral momentum $$\frac{d}{dx}\int_0^h \left(1 - \frac{u}{U}\right)\frac{u}{U}\,dy = \frac{\tau_w}{\rho U^2}$$

Power-law profile $$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

Wall shear stress $$\tau_w = 0.0296\left(\frac{\nu}{Ux}\right)^{1/5}\rho U^2$$

Boundary layer thickness $$\frac{d\ \delta}{dx} = \frac{72}{7} \times 0.0296\left(\frac{\nu}{U}\right)^{1/5} x^{-1/5}$$

Heat transfer analogy

Boundary layer thickness

$$\frac{\delta}{x} = 0.381\,\mathrm{Re}_x^{-1/5} - 10,256\,\mathrm{Re}_x^{-1}$$

Friction coefficient

$$\frac{C_f}{2} = \frac{\tau_w}{\rho U^2} = 0.0296\left(\frac{\nu}{Ux}\right)^{1/5}$$

Analogy

$$\frac{C_f}{2} = \mathrm{St}_x\,\mathrm{Pr}^{2/3}$$

$$\mathrm{St}_x\,\mathrm{Pr}^{2/3} = 0.0296\,\mathrm{Re}^{-1/5}$$

or

$$\mathrm{Nu}_x = 0.0296\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re} < 10^7$$

Average Nu for *L* >>*x*$_c$

$$\overline{\mathrm{Nu}}_L = 0.037\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

Heat transfer in pipe flow

Energy equation

$$u\frac{\partial T}{\partial z}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Bulk temperature

$$\overline{T}=\frac{\int \rho c T u dA}{\int \rho c u dA}=\frac{2}{\overline{u}R^2}\int_0^R uTr\;dr$$

Developed temp. profile

$$\theta=\frac{T-T_w}{\overline{T}-T_w}\neq\theta(z),\qquad \frac{\partial\theta}{\partial z}=0\quad\rightarrow\quad\frac{\partial\theta}{\partial r}\neq f(z)$$

Wall heat flux

$$q_w=h\left(T_w-\overline{T}\right)=-k\left.\frac{\partial T}{\partial y}\right|_{y=o}=k\left.\frac{\partial T}{\partial r}\right|_{r=R}$$

Constant h

$$\frac{h}{k}=\frac{1}{T_w-\overline{T}}\left.\frac{\partial T}{\partial r}\right|_{r=R}=-\left.\frac{\partial\theta}{\partial r}\right|_{r=R}=\mathrm{const}$$

Asymptotic solution

Energy equation

$$2\bar{u}\left(1-\left(\frac{r}{R}\right)^2\right)\frac{d\bar{T}}{dz}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

boundary cond.

$$T(R,z)=T_w,\qquad \left.\frac{\partial T}{\partial r}\right|_{r=0}=0$$

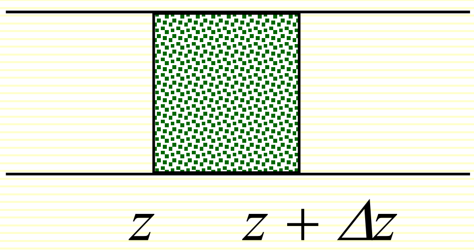
Solution

$$T=\frac{2\bar{u}R^2}{\alpha}\left(\frac{d\bar{T}}{dz}\right)\left[\frac{1}{4}\left(\frac{r}{R}\right)^2-\frac{1}{16}\left(\frac{r}{R}\right)^4\right]+C_1\ln r+C_2$$

$$T=T_w+\frac{\bar{u}R^2}{2\alpha}\left(\frac{d\bar{T}}{dz}\right)\left[\left(\frac{r}{R}\right)^2-\frac{1}{4}\left(\frac{r}{R}\right)^4-\frac{3}{4}\right]$$

Nusselt number

Heat balance



$$\dot{m}c\bar{T}\Big|_{z} - \dot{m}c\bar{T}\Big|_{z+\Delta z} + q_w 2\pi R\,\Delta z = 0; \qquad \dot{m} = \rho\bar{u}\,\pi R^2$$

$$\frac{d\bar{T}}{dz} = \frac{2\pi R}{\dot{m}c}q_w \qquad\qquad \frac{d\bar{T}}{dz} = \frac{2h}{\rho\bar{u}cR}\left(T_w - \bar{T}\right)$$

$$\bar{T} = \frac{2}{\bar{u}R^2}\int_0^R uTr\,dr = T_w - \frac{11}{48}\frac{\bar{u}R^2}{\alpha}\frac{d\bar{T}}{dz}$$

$$\bar{T} - T_w = \frac{11}{48}\frac{\bar{u}R^2}{\alpha}\frac{2h}{\rho\bar{u}cR}\left(\bar{T} - T_w\right) = \frac{11}{48}\frac{hD}{k}\left(\bar{T} - T_w\right)$$

Nusselt No. for $q_w = \mathrm{const}$ $\qquad \mathrm{Nu}_D = \dfrac{hD}{k} = 4.364$

Nusselt No. for $T_w = \mathrm{const}$ $\qquad \mathrm{Nu}_D = \dfrac{hD}{k} = 3.658$

Turbulent heat transfer in pipes

Turbulent momentum transfer

$$\frac{\tau_w}{\rho} = (\nu + \varepsilon_M)\left.\frac{\partial u}{\partial r}\right|_{r=R} \quad (1)$$

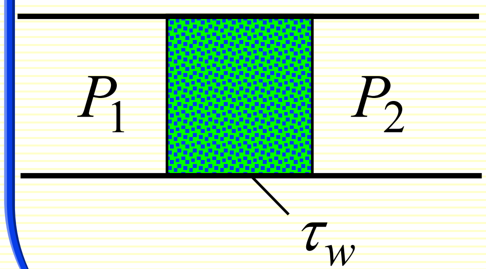
Turbulent heat transfer

$$\frac{q_w}{\rho c} = -(\alpha + \varepsilon_H)\left.\frac{\partial T}{\partial r}\right|_{r=R} \quad (2)$$

Divide (1) by (2) for Pr=1, $\varepsilon_M = \varepsilon_H$

$$\frac{q_w}{c\ \tau_w} = -\frac{\partial T}{\partial u} \approx -\frac{\Delta T}{\Delta u} = \frac{T_w - \overline{T}}{\overline{u}} \quad (3)$$

Force balance



$$\tau_w \cdot \pi D L = \Delta P \cdot \frac{\pi D^2}{4} \quad \rightarrow \quad \tau_w = \frac{1}{4}\frac{\Delta P}{L/D}$$

$$\Delta P = f\,\frac{L}{D}\,\frac{\rho \overline{u}^2}{2} \quad \rightarrow \quad \tau_w = \frac{f}{8}\rho \overline{u}^2 \quad (4)$$

Turbulent heat transfer analogy

Combine (3) and (4)

$$\frac{q_w}{T_w - \overline{\overline{T}}} \frac{1}{\rho \bar{u} c} = \frac{f}{8} \quad \rightarrow \quad \frac{h}{\rho \bar{u} c} = \frac{f}{8}$$

Reynolds analogy

$$\mathrm{St} = \frac{f}{8} \qquad \mathrm{Pr} = 1$$

Reynolds-Colburn analogy

$$\frac{f}{8} = \mathrm{St} \cdot \mathrm{Pr}^{2/3} \equiv j_H \qquad \mathrm{Nu}_D = \frac{f}{8} \mathrm{Re} \cdot \mathrm{Pr}^{1/3}$$

$$f = 0.316\,\mathrm{Re}^{-0.25} \quad \rightarrow \quad \mathrm{Nu}_D = 0.0395\,\mathrm{Re}^{0.75}\,\mathrm{Pr}^{1/3}$$

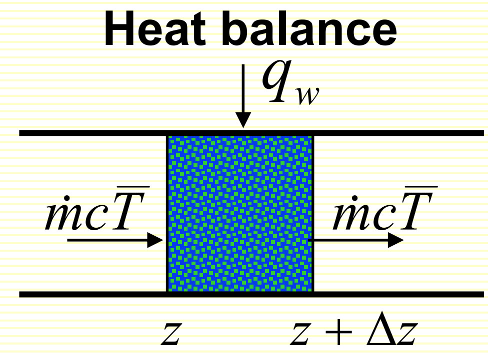
$$f = 0.184\,\mathrm{Re}^{-0.2} \quad \rightarrow \quad \mathrm{Nu}_D = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

For non-circular pipes

$$D_H = \frac{4A}{P}$$

Energy balance in pipes

$$\dot{m}c\bar{T}\Big|_z - \dot{m}c\bar{T}\Big|_{z+\Delta z} + q_w 2\pi R\,\Delta z = 0; \qquad \dot{m} = \rho \bar{u} \pi R^2$$



$$(1) \quad \frac{d\bar{T}}{dz} = \frac{2\pi R}{\dot{m}c} q_w \qquad\qquad (2) \quad \frac{d\bar{T}}{dz} = \frac{2\pi R}{\dot{m}c} h\left(T_w - \bar{T}\right)$$

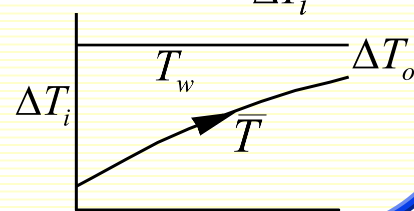
For $q_w = \text{const}$ with $T(0) = \bar{T}_i$ Eq.(1) yields

$$\bar{T}(z) = \bar{T}_i + \frac{2\pi R q_w}{\dot{m}c} z \qquad\qquad \dot{Q} = q_w A = 2\pi r L q_w$$

For $T_w = \text{const}$ with $\Delta T = T_w - \bar{T}$ and $d(\Delta T) = -d\bar{T}$ Eq. (2) yields

$$\frac{d(\Delta T)}{\Delta T} = -\frac{2\pi R}{\dot{m}c} h \; dz \qquad \ln\frac{\Delta T_o}{\Delta T_i} = \ln\frac{T_w - \bar{T}_o}{T_w - \bar{T}_i} = -\frac{2\pi R h}{\dot{m}c} L \qquad \dot{m}c = \frac{2\pi R h L}{\ln\frac{\Delta T_o}{\Delta T_i}}$$

$$\dot{Q} = \dot{m}c\left(T_o - T_i\right) = 2\pi R h L \frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}} \qquad\qquad \dot{Q} = hA\Delta T_{LM}$$



Empirical correlations

Dittus-Boelter correlation

$$\mathrm{Nu}_D = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^n$$

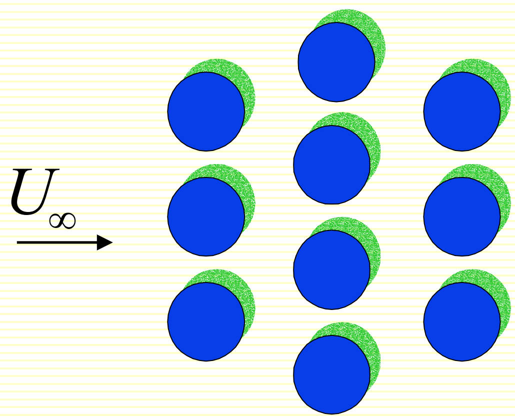
$$n = 0.4 \;\; \text{heating} \qquad n = 0.3 \;\; \text{cooling}$$

Convection to tube banks

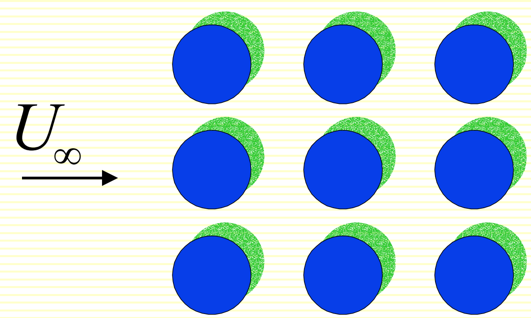
$$\frac{hD}{k} = C\left(\frac{U_{\max} D}{\nu}\right)^n \mathrm{Pr}^{1/3}$$

$$\text{In line}: \qquad 0.05 < C < 0.5, \quad 0.55 < n < 0.8$$

$$\text{Staggered}: \quad 0.2 < C < 0.6, \quad 0.55 < n < 0.65$$

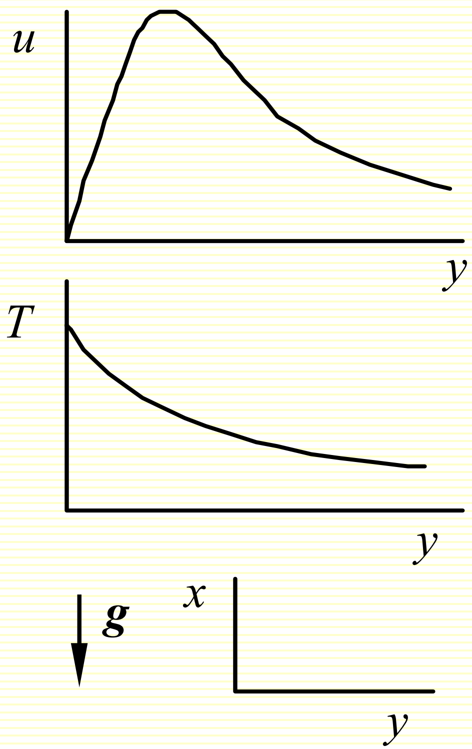


Staggered



In line

Free convection



$$\rho\left(u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}-\rho g+\mu\frac{\partial^2 u}{\partial y^2} \qquad \textbf{(1)}$$

$$\text{at} \quad y\rightarrow\infty \qquad 0=-\frac{\partial p}{\partial x}-\rho_\infty g \qquad \textbf{(2)}$$

Combining (1) and (2)

$$\rho\left(u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}\right)=(\rho_\infty-\rho)g+\mu\frac{\partial^2 u}{\partial y^2} \qquad \textbf{(3)}$$

Expansion coefficient

$$\beta=\frac{1}{\upsilon}\left(\frac{\partial\upsilon}{\partial T}\right)_p\approx\frac{1}{\upsilon_\infty}\frac{\upsilon-\upsilon_\infty}{T-T_\infty}=\frac{\rho_\infty-\rho}{\rho(T-T_\infty)}$$

$$\rho_\infty-\rho=\rho\beta(T-T_\infty)$$

Free convection equations

Momentum
$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=g\beta\left(T-T_{\infty}\right)+\nu\frac{\partial^2 u}{\partial y^2}, \qquad \left(T-T_{\infty}\right)=\Delta T\,\theta$$

Energy
$$u\frac{\partial \theta}{\partial x}+\upsilon\frac{\partial \theta}{\partial y}=\alpha\frac{\partial^2 \theta}{\partial y^2} \qquad \text{where} \qquad \theta=\frac{T-T_{\infty}}{T_w-T_{\infty}}=\frac{T-T_{\infty}}{\Delta T}$$

Integral momentum equation

$$\frac{d}{dx}\int_0^{\delta}u^2dy=g\beta\Delta T\int_0^{\delta}\theta dy-\nu\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

Integral energy equation

$$\frac{d}{dx}\int_0^{\delta}u\theta dy=-\alpha\left.\frac{\partial \theta}{\partial y}\right|_{y=0}$$

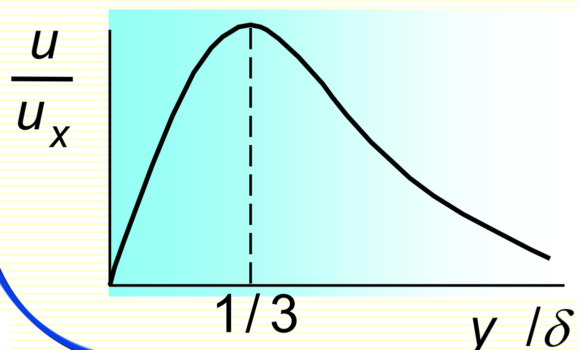
Velocity and temperature profiles

Boundary cond. $\theta(0)=1, \quad \theta(\delta)=0, \quad \left.\dfrac{\partial\theta}{\partial y}\right|_{y=\delta}=0$

Temperature profile $\theta = a + by + cy^2 \quad \rightarrow \quad \theta = (1 - y/\delta)^2$

Velocity profile $u = a + by + cy^2 + dy^3$

Boundary cond. $u(0)=0, \quad u(\delta)=0, \quad \left.\dfrac{\partial u}{\partial y}\right|_{\delta}=0 \quad \left.\dfrac{\partial^2 u}{\partial y^2}\right|_{y=0} = -\dfrac{g\beta\Delta T}{\nu}$



Velocity profile

$$\frac{u}{u_x} = \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2, \quad u_x = u_0\frac{g\beta\delta^2\Delta T}{4\nu}, \quad u_{\max} = \frac{4}{27}u_x$$

Free convection integral solution

Integral momentum equation yields

$$\frac{1}{105}\frac{d}{dx}\left(u_x\delta\right) = \frac{1}{3} g\ \beta\Delta T\ \delta - \nu\frac{u_x}{\delta}$$

Integral energy equation yields

$$\frac{\Delta T}{30}\frac{d}{dx}\left(u_x\delta\right) = 2\alpha\frac{\Delta T}{\delta}$$

Let $u_x = C_1 x^m, \qquad \delta = C_2 x^n$

Get

$$\frac{(27/4)^2}{105} C_1^2 C_2 (2m+n) x^{2m+n-1} = \frac{g\beta\Delta T C_2}{3} x^n - \frac{27}{4}\frac{C_1}{C_2}\nu x^{m-n}$$

$$\frac{27}{4}\frac{C_1 C_2 (m+n)}{30} x^{m+n-1} = \frac{2\alpha}{C_2} x^{-n}$$

$$2m+n-1 = n = m-n, \qquad m+n-1 = -n$$

$$m = \frac{1}{2}, \qquad n = \frac{1}{4}$$

Boundary layer thickness

$$C_1 = \frac{320\nu}{27^2\sqrt{15}}\left(\frac{20}{21}+\Pr\right)^{-1/2}\left(\frac{g\beta\Delta T}{\nu^2}\right)^{1/2}$$

$$C_2 = 240^{1/4}\left(\frac{20}{21}+\Pr\right)^{1/4}\left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/4}\Pr^{-1/2}$$

The Grashof number

$$\mathrm{Gr}_x = \frac{g\,\beta\Delta T\,x^3}{\nu^2}$$

Since

$$\delta = C_2\ \ x^n \quad\rightarrow\quad \frac{\delta}{x} = \frac{C_2}{x^{3/4}}$$

Therefore

$$\frac{\delta}{x} = 3.93\,\Pr^{-1/2}\left(\frac{0.952+\Pr}{\mathrm{Gr}_x}\right)^{1/4}$$

The heat transfer coefficient

Heat flux

$$q_w = h\ \Delta T = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -k\ \Delta T\frac{\partial \theta}{\partial y}\bigg|_{y=0} = -k\ \Delta T\left(-\frac{2}{\delta}\right)$$

Nusselt No.

$$h = \frac{2k}{\delta}, \quad \rightarrow \quad \mathrm{Nu}_x = \frac{0.508\ \mathrm{Pr}^{1/2}\mathrm{Gr}_x^{1/4}}{(0.952+\mathrm{Pr})^{1/4}}$$

Average values

$$\bar{h} = \frac{1}{L}\int_0^L h\ dx = \frac{4}{3}h\bigg|_{x=L}$$

$$\overline{\mathrm{Nu}} = \frac{4}{3}\mathrm{Nu}\bigg|_{x=L}$$

For air

$$\beta = \frac{1}{T} \approx \frac{1}{T_\infty} \quad \rightarrow \quad \mathrm{Nu}_x = 0.378\ \mathrm{Gr}_x^{1/4}$$

Empirical correlations

General form $\overline{\mathrm{Nu}} = C(\mathrm{Gr}_L \ \mathrm{Pr})^m = C \ \mathrm{Ra}^m$

The Rayleigh No. $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g \, \beta \Delta T \, L^3}{\alpha \nu}$

Equations for air

	Laminar flow $\mathrm{Ra} < 10^9$	**Turbulent flow** $\mathrm{Ra} > 10^9$
Vertical plate or cylinder	$h = 1.42\left(\dfrac{\Delta T}{L}\right)^{1/4}$	$h = 1.31(\Delta T)^{1/3}$
Horizontal cylinder	$h = 1.32\left(\dfrac{\Delta T}{d}\right)^{1/4}$	$h = 1.24(\Delta T)^{1/3}$
Horizontal heated plate ↑	$h = 1.32\left(\dfrac{\Delta T}{L}\right)^{1/4}$	$h = 1.52(\Delta T)^{1/3}$

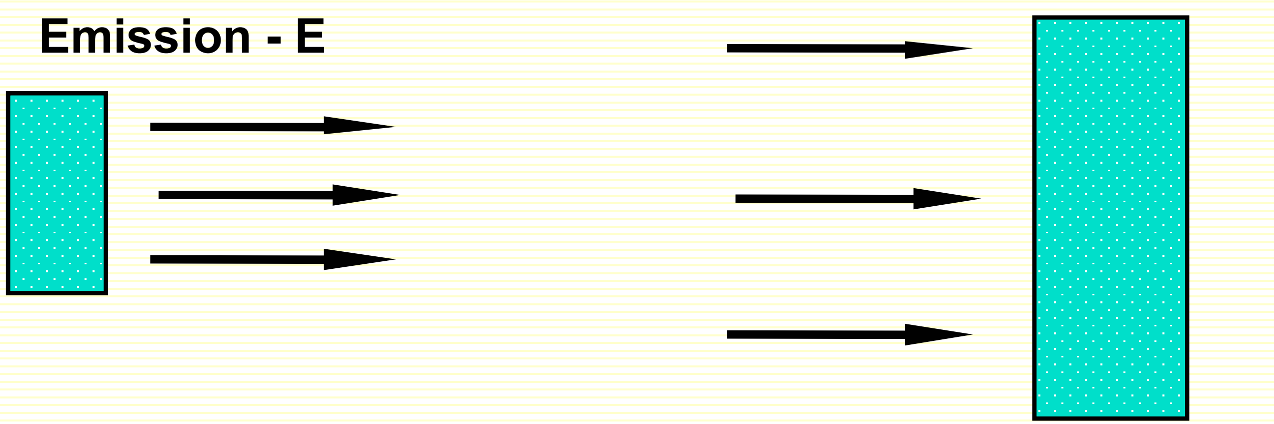
Radiation

Thermal radiation $0.01\,\mu m < \lambda < 10\,\mu m$

Visible light $0.35\,\mu m < \lambda < 0.78\,\mu m$

Speed of light $c = \lambda \cdot \nu$ where $c = 3 \times 10^{8}\,\mathrm{m/s}$

Radiating and irradiated bodies



Black body radiation

Black body spectral emissive power - Planck law

$$E_{b\lambda} = \frac{2\pi h c^2}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)} \left[\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mu\mathrm{m}}\right]$$

Planck's constant

$$h = 6.626176 \times 10^{-34}\,\mathrm{J \cdot s}$$

$$\frac{E_{b\lambda}}{T^5} = \frac{c_1}{(\lambda T)^5 \left(e^{c_2/\lambda T} - 1\right)}$$

Black body total emissive power - Stefan-Boltzmann law

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda = \sigma T^4 \left[\frac{W}{m^2}\right]$$

where

$$\sigma = 5.669 \times 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4} = 0.1714 \times 10^{-8}\,\frac{\mathrm{BTU}}{\mathrm{hr} \cdot \mathrm{ft}^2 \cdot {}^{\mathrm{o}}\,\mathrm{R}^4}$$

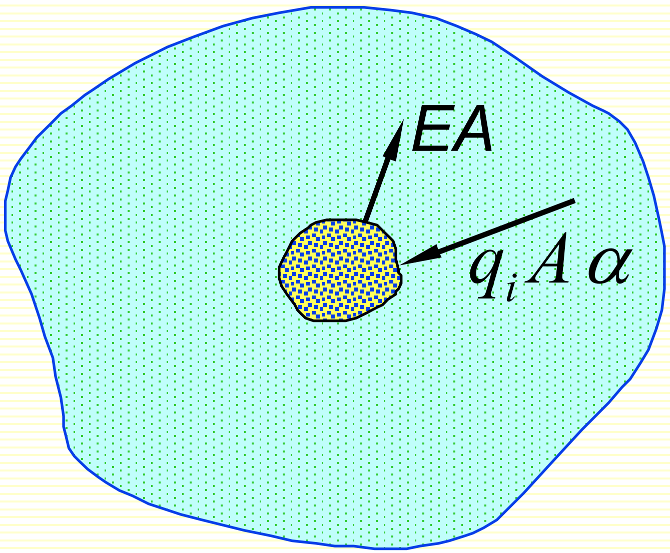
Heat transfer rate

$$\dot{Q}_b = E_b A = \sigma T^4 A\,[\mathrm{W}]$$

Irradiated body

In general $\alpha + \rho + \tau = 1$

Opaque body: $\alpha + \rho = 1$ **Black body:** $\alpha = 1$



In general: $EA = q_i A\,\alpha$

For black body: $E_b A = q_i A \cdot 1$

$\therefore \quad \alpha = E / E_b$

Define emissivity $\varepsilon = E / E_b$

Kirchhoff's Law $\varepsilon = \alpha$

Definition of gray body $\varepsilon_\lambda = E_\lambda / E_{b\lambda} = \varepsilon = \text{const}$

For gray body $\dot{Q} = \varepsilon A\,\sigma T^4$

Radiation shape factor

$F_{m\ n}$ - **Fraction of radiation energy leaving *m* and reaching *n***

Net radiation between black bodies $\dot{Q}_{1-2} = A_1 F_{12} E_{b1} - A_2 F_{21} E_{b2}$

For $T_1 = T_2$ $\dot{Q}_{1-2} = 0 \quad E_{b1} = E_{b2}$

$$\therefore A_1 F_{12} = A_2 F_{21}$$

$$\dot{Q}_{1-2} = A_1 F_{12}\left(E_{b1} - E_{b2}\right) = A_2 F_{21}\left(E_{b1} - E_{b2}\right)$$

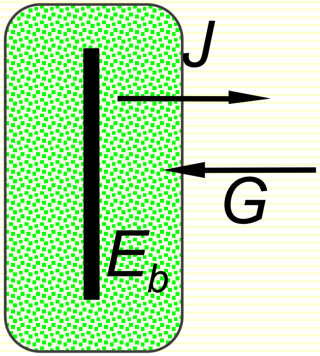
Shape factor relations

$$\sum_{j=1}^{n} F_{ij} = 1.0$$

$$F_{1-2,3} = F_{1-2} + F_{1-3}$$

$$F_{12} = \frac{A_2}{A_1} F_{21}$$

Radiation between gray bodies



***J* - radiosity,** ***G* - irradiation**

$$q = \frac{\dot{Q}}{A} = J - G$$

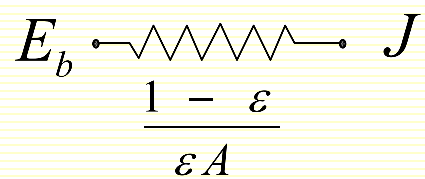
$$J = \varepsilon E_b + \rho G = \varepsilon E_b + (1-\varepsilon)G$$

Irradiation, i.e., total radiation arriving at surface

$$G = \frac{J - \varepsilon E_b}{(1-\varepsilon)} \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

Net heat transfer from gray body

$$\dot{Q} = \frac{E_b - J}{(1-\varepsilon)/\varepsilon A}$$

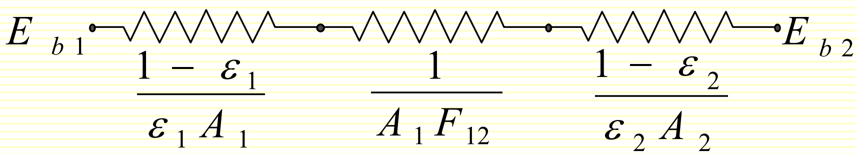


Radiation heat transfer between two gray bodies

$$\dot{Q}_{1-2} = \frac{J_1 - J_2}{1/A_1F_{12}} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Gray body relations

Radiation between two gray bodies



$$\dot{Q}_{1-2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

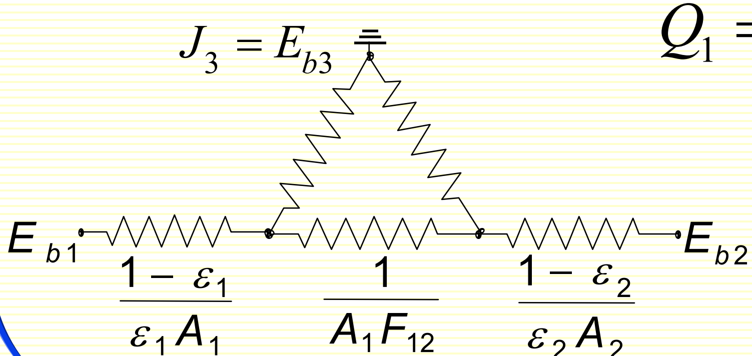
Special case:

$$F_{12} = F_{21} = 1$$

$$A_1 = A_2$$

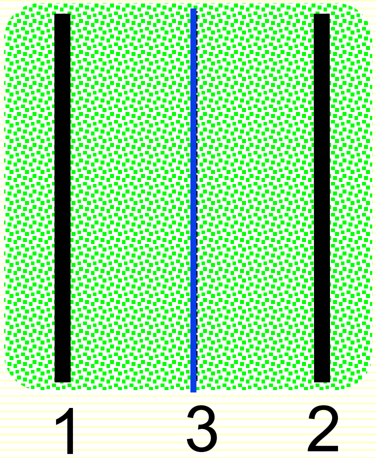
$$\dot{Q}_{1-2} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

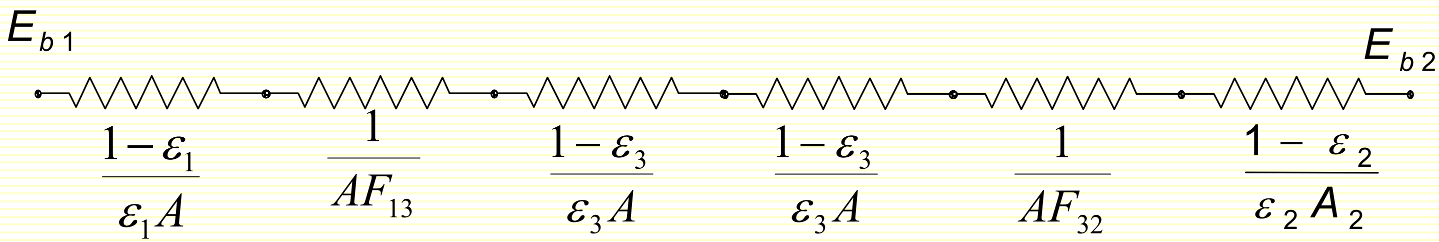
Reradiating surface (3)



$$\dot{Q}_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \dfrac{1}{\left(1/A_1 F_{13}\right) + \left(1/A_2 F_{23}\right)}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Radiation shields





$$q=\frac{\dot{Q}}{A}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A}+\frac{1}{AF_{13}}+2\frac{1-\varepsilon_3}{\varepsilon_3 A}+\frac{1}{AF_{32}}+\frac{1-\varepsilon_2}{\varepsilon_2 A}}$$

For $F_1=F_2=F_3=1$ $\longrightarrow$ $q=\frac{\dot{Q}}{A}=\frac{1}{2}\cdot\frac{\sigma\left(T_1^4-T_2^4\right)}{2/\varepsilon-1}$

$\varepsilon_1=\varepsilon_2=\varepsilon_3=\varepsilon$

For *n* shields $\longrightarrow$ $q=\frac{\dot{Q}}{A}=\frac{1}{n+1}\cdot\frac{\sigma\left(T_1^4-T_2^4\right)}{2/\varepsilon-1}$

Heat Exchangers - Classification

a. **Classification by type:**
   Regenerators
   Recuperators

b. **Classification by flow character:**
   Single phase: liquid–liquid, liquid–gas, gas–gas.
   Two-phase: boilers, reboilers, evaporators, condensers

c. **Classification by shape:**
   double pipe, shell-and-tube, plate h.e., air radiator, stirred tank heat exchanger.

Thermal design problems

Problem #1:

Given entrance temperatures of the two streams,
given one exit temperature;
find heat-transfer area, *A*.

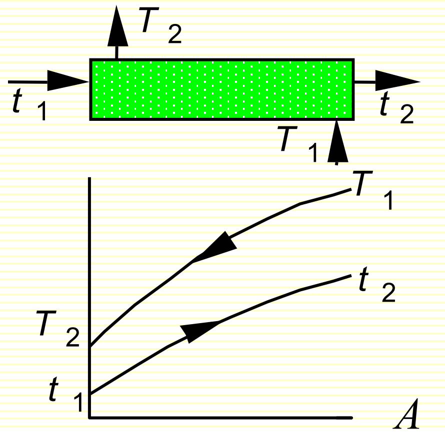
Problem #2:

Given entrance temperatures of the two streams,
given the heat-transfer area, *A*;
find the exit temperatures of the two streams.

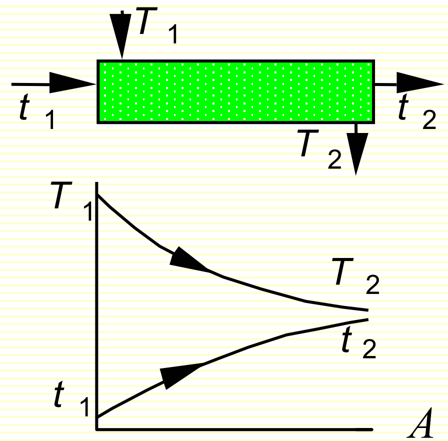
Design algorithm

Input	Calculation	Results
1. Flowrates	A. Calculation of size and geometry	Exit temperatures for given area
2. Temperatures	B. Heat transfer correlations (*h*)	Heat transfer area for given thermal load
3. Pressures	C. Pressure drop correlations (*h*)	Pressure drops
4. Shape of h.e.		
5. Properties of fluids		
6. Fouling factors		

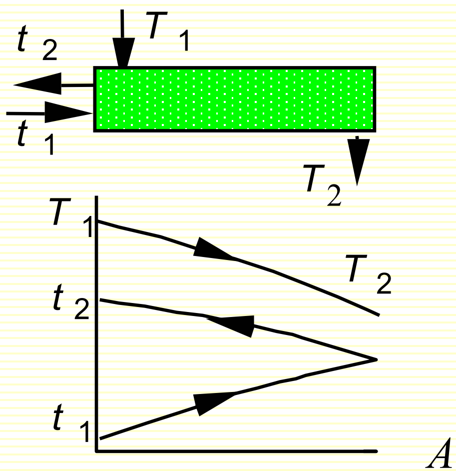
Flow configurations

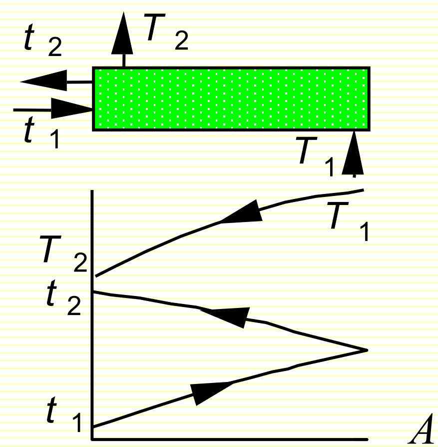


Counter-current



Co-current





Heat exchanger with 1 shell and 2 tube passes

Overall heat transfer coefficient

Plane wall $\dot{Q} = UA\Delta T$

$$\frac{1}{U} = \frac{1}{h_A} + R_{FA} + \frac{\Delta x}{k} + R_{FB} + \frac{1}{h_B}$$

Cylindrical wall $\dot{Q} = U_o A_o \Delta T$

$$\frac{1}{U_o} = \left(\frac{D_o}{D_i}\right)\frac{1}{h_i} + \left(\frac{D_o}{D_i}\right)R_{Fi} + \frac{D_o}{2k}\ln\left(\frac{D_o}{D_i}\right) + R_{Fo} + \frac{1}{h_o}$$

Typical correlation for *h*

$$\frac{hD}{k} = 0.023\left(\frac{D\overline{v}}{\nu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{1/3}$$

Thermal analysis

1. **Mass balance** $\dot{m} = \rho \bar{u} A_x N$

2. **Heat balance** $\dot{Q} = \dot{m}_c (h_2 - h_1)_c = \dot{m}_h (h_1 - h_2)_h$

For $h = cT$ $\dot{Q} = \dot{m}_c c_c (t_2 - t_1) = \dot{m}_h c_h (T_1 - T_2)$

3. **Rate equation**

$$\dot{Q} = U_o A_o \Delta T_{LM} = U_o A_o \frac{\Delta T_A - \Delta T_B}{\ln \dfrac{\Delta T_A}{\Delta T_B}}$$

Rate equation for multi-pass etc.

$$\dot{Q} = U_o A_o \Delta T_{LM} \cdot F = U_o A_o \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln \dfrac{(T_1 - t_2)}{(T_2 - t_1)}} \cdot F$$

HEAT TRANSFER WITH PHASE CHANGE

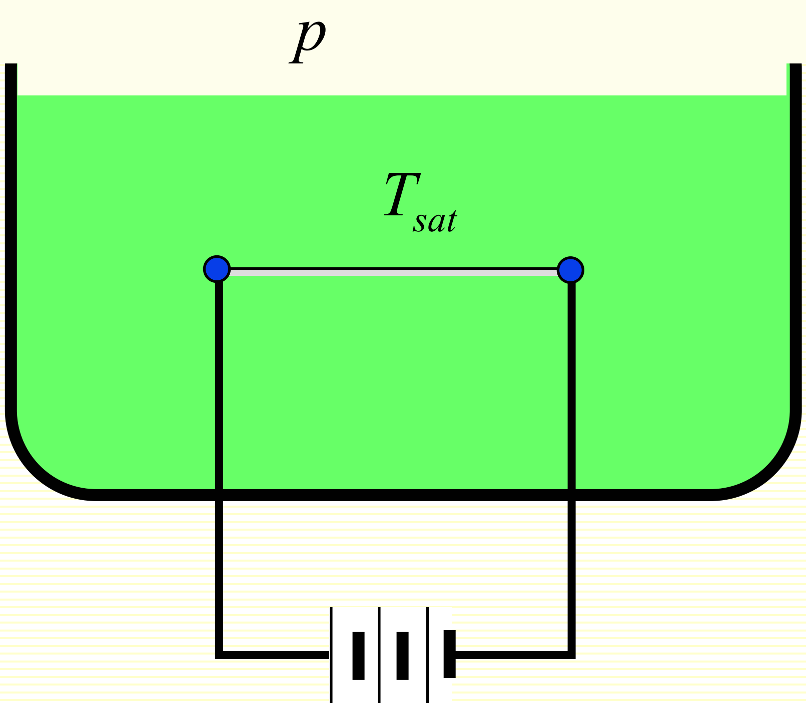
BOILING

The Pool Boiling Curve
The Boiling Process
Nucleate Boiling Correlations
Critical Heat Flux Correlations
Film Boiling Correlations
Forced Convection Boiling
Flow Boiling Correlations

CONDENSATION

Dropwise vs. Filmwise Condensation
Condensation on Vertical Surfaces
Condensation on Horizontal Tubes
Condensation Inside Tubes

Pool boiling



Nukiyama's Experiment

Given q find T_w

Vapor-liquid equilibrium

$$T_G = T_L$$

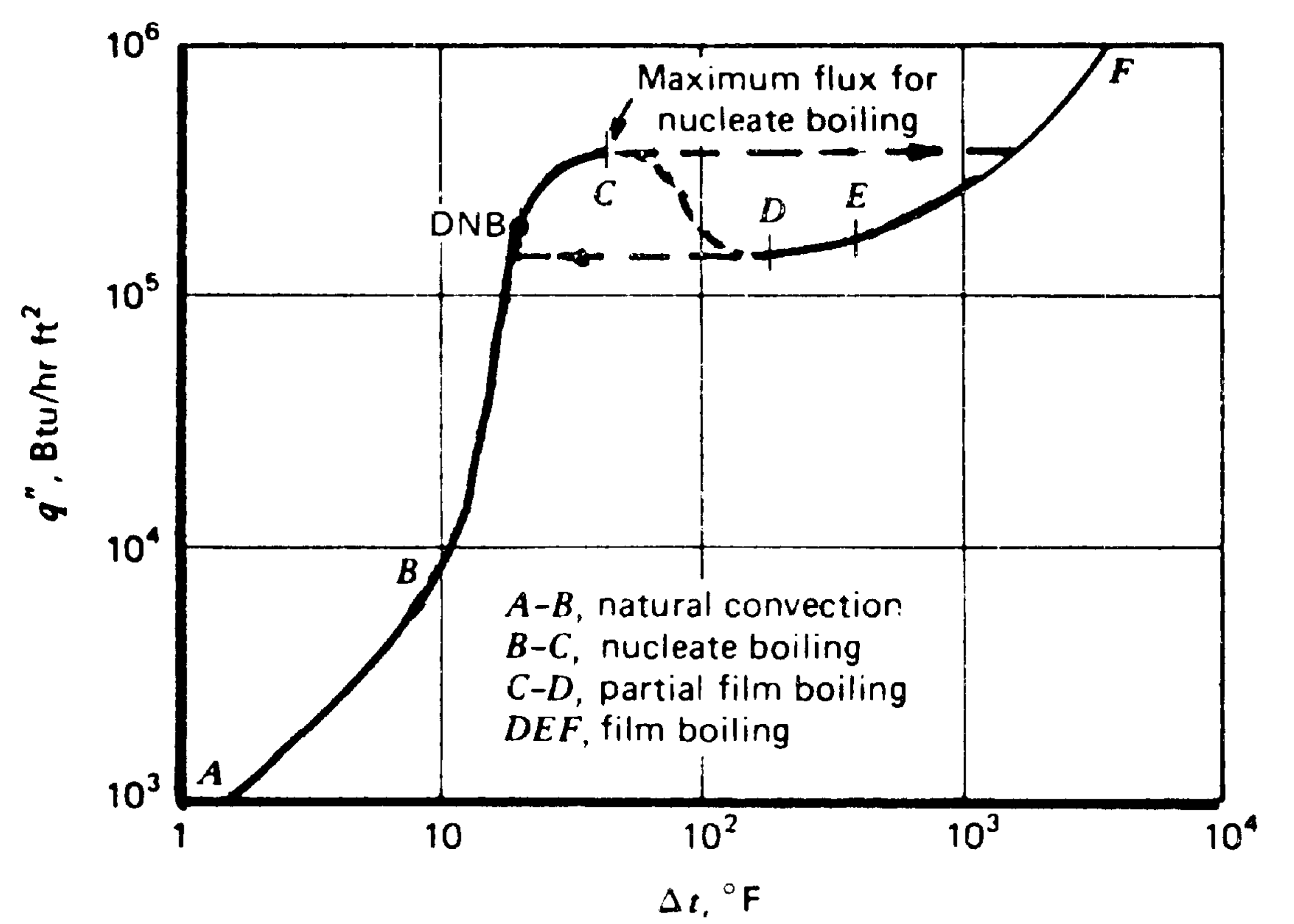
$$\mu_G = \mu_L$$

$$p_G = p_L + \frac{2\sigma}{r}$$

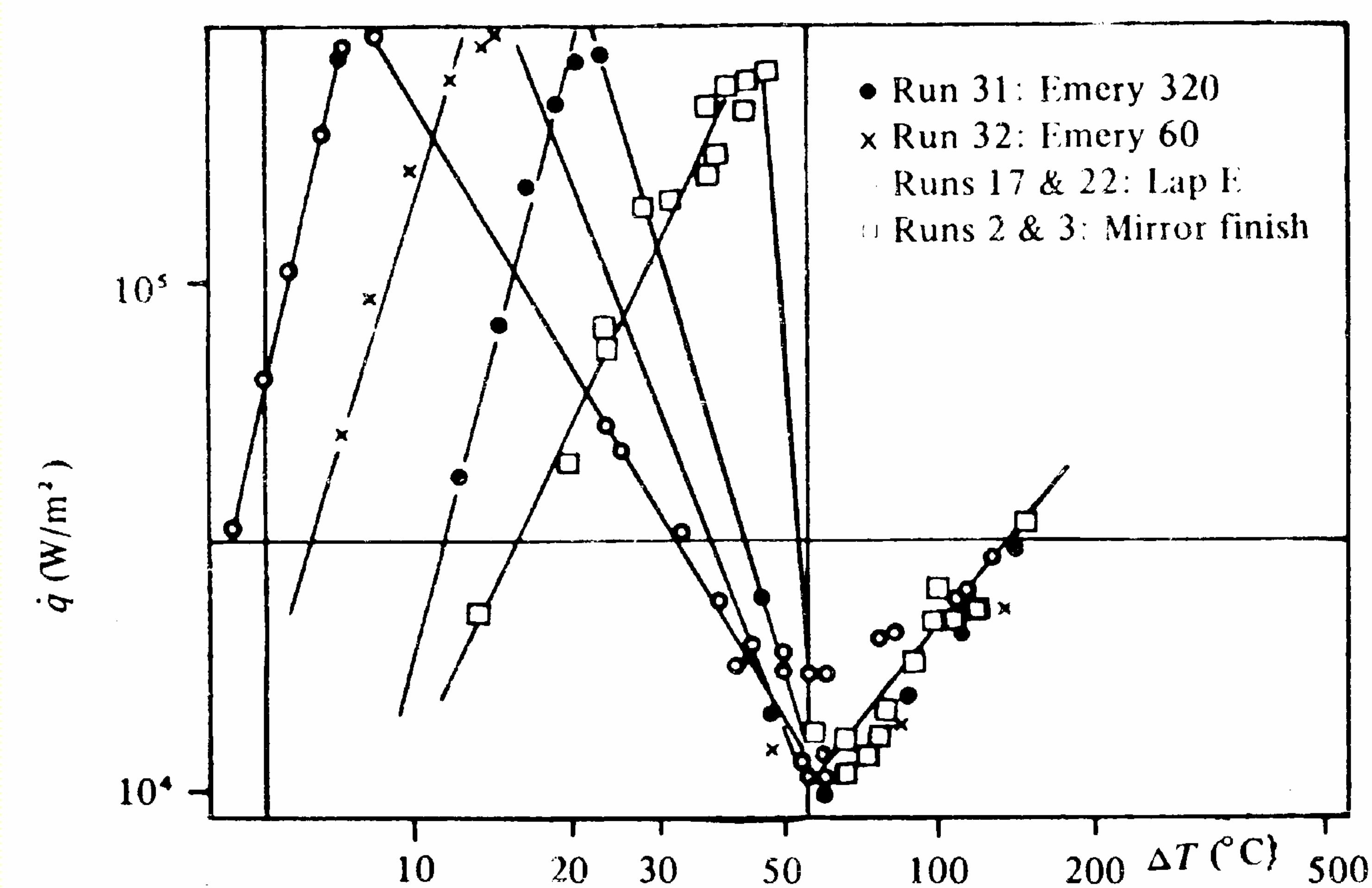
Temperature that sustains a bubble of radius *r*

$$T = T_{sat}(p_L)\left[1 + \frac{\upsilon_{LG}}{h_{LG}}\frac{2\sigma}{r}\frac{\rho_L}{\rho_L - \rho_G}\right]$$

Boiling Curve



Effect of surface roughness on boiling



Boiling heat transfer coefficient

Heat transfer coefficient $$h_b = \frac{q''}{T_w - T_{sat}}$$

Nusselt number $$\mathrm{Nu} = \frac{h_b D_B}{k_L} = \frac{q'' D_B}{(T_w - T_{sat})k_L} \quad ; \quad \mathrm{Nu} = f(\mathrm{Re}_B, \mathrm{Pr}_L)$$

Reynolds number $$\mathrm{Re} = \frac{G_B D_B}{\mu_L}$$ **where** $$G_B = \frac{\pi}{6} D_B^3 \rho_B f n$$

$$D_B = 0.0148\,\beta \left[\frac{2\sigma}{g(\rho_L - \rho_G)}\right]^{1/2}$$

Heat flux $$q \propto (T_w - T_{sat})^n$$ **where** $$n \approx 3$$

Pool boiling correlations

Rohsenow (1952)

$$q = \mu_L h_{LG}\left[\frac{g(\rho_L-\rho_G)}{\sigma}\right]^{1/2}\left[\frac{c_L(T_w-T_{sat})}{h_{LG}\,\mathrm{Pr}_L^{1.7}C_{sf}}\right]^3$$

Critical heat flux

Zuber (1959)

$$\frac{q_{\max}}{\rho_G h_{LG}} = 0.149\left[\frac{\sigma(\rho_L-\rho_G)g}{\rho_G^2}\right]^{1.4}\left(\frac{\rho_L+\rho_G}{\rho_L}\right)^{1/2}$$

Kutateladze (1952)

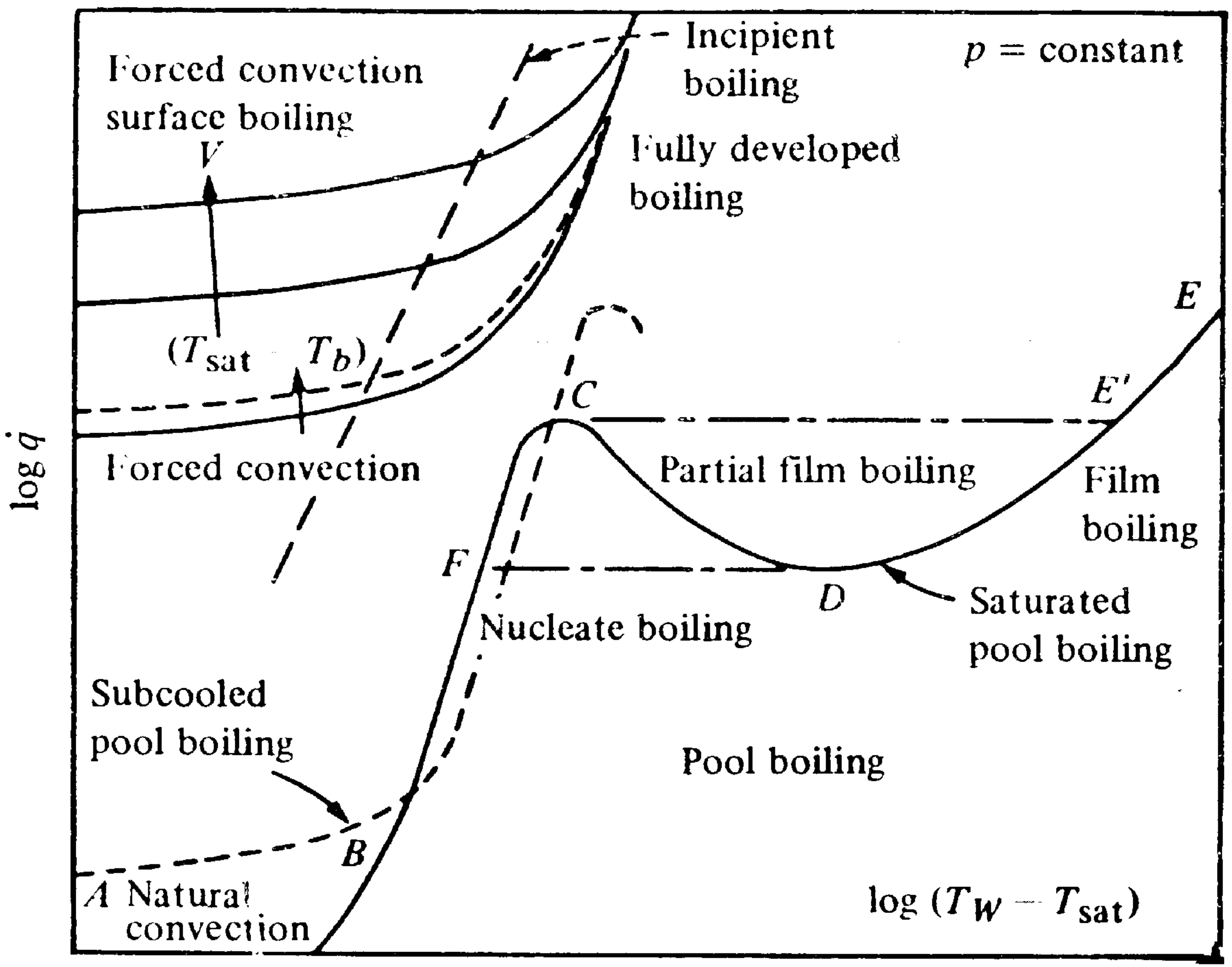
$$\frac{q_{\max}}{\rho_G h_{LG}} = 0.16\left[\frac{\sigma(\rho_L-\rho_G)g}{\rho_G^2}\right]^{1/4}$$

Film boiling – minimum heat flux

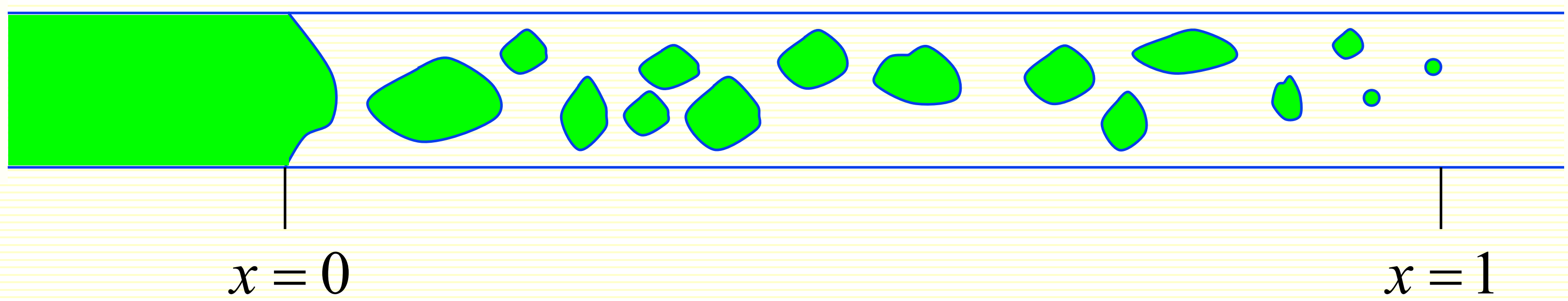
Zuber & Tribus (1958)

$$\dot{q}_{\min} = 0.09\rho_{Gf}h_{LG}\left[\frac{g(\rho_L-\rho_G)}{\rho_L+\rho_G}\right]^{1/2}\left[\frac{\sigma}{g(\rho_L-\rho_G)}\right]^{1/4}$$

Forced convection boiling



Convective boiling & quality

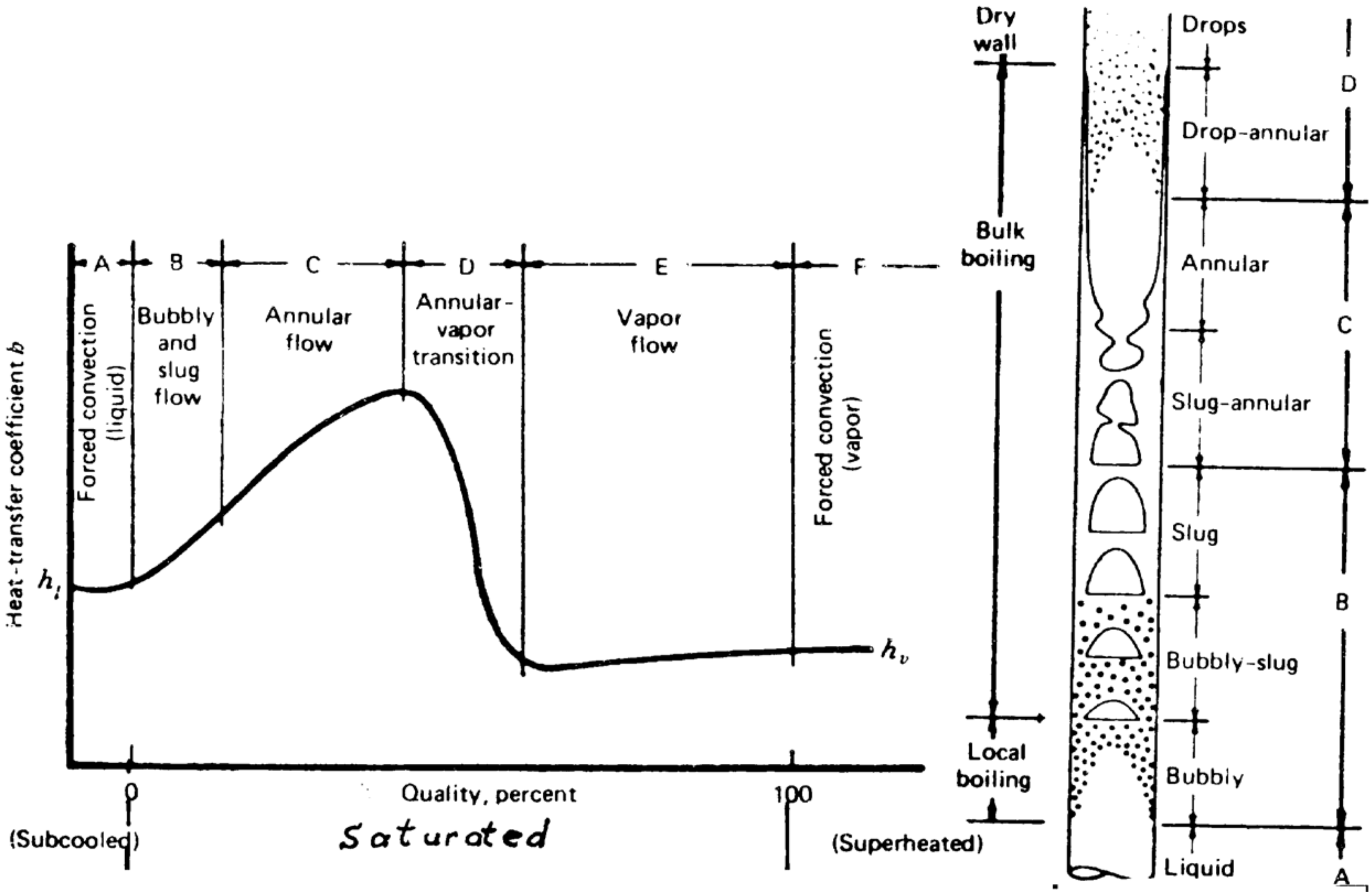


Definition of quality $x = m_G / m$

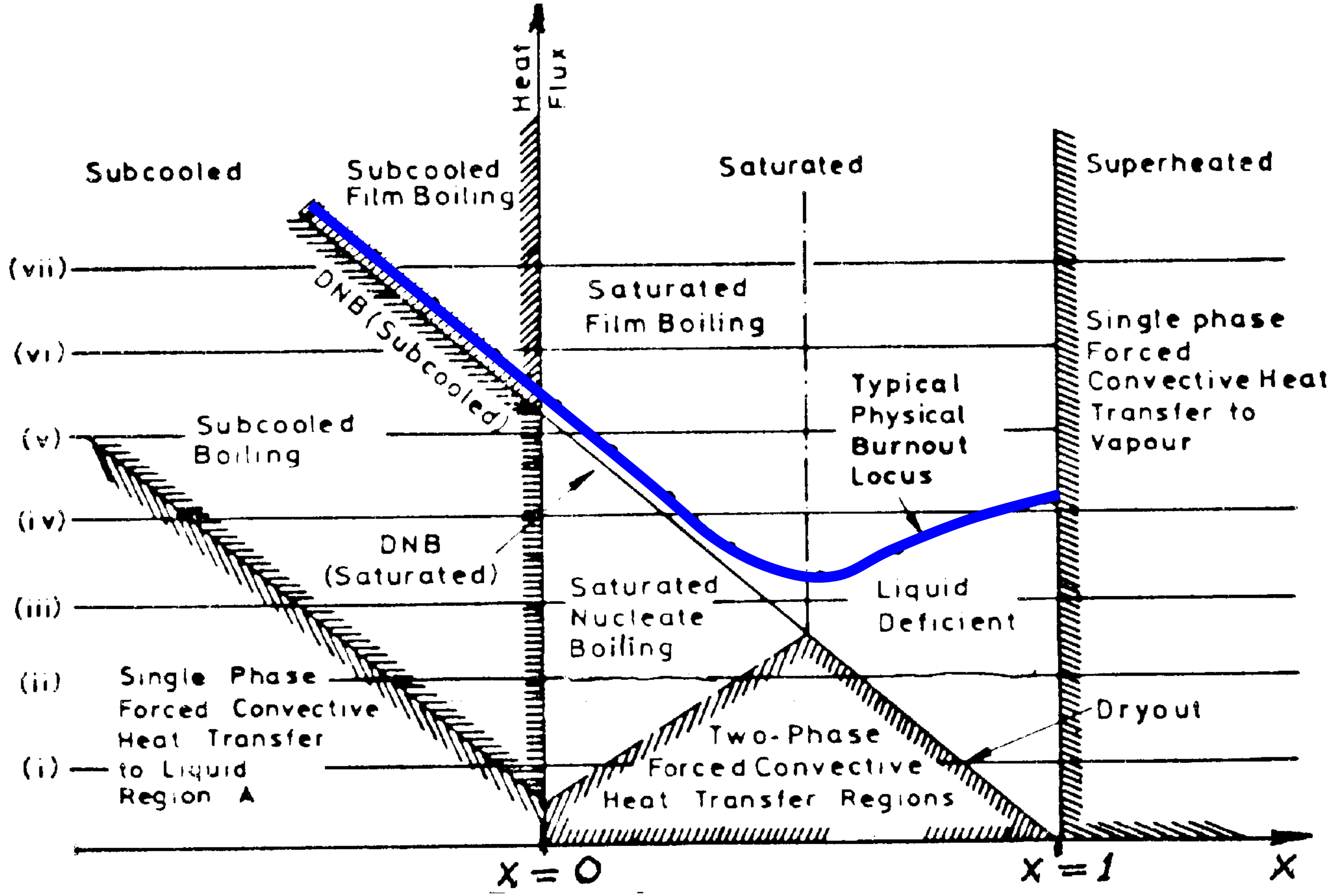
At equilibrium $h = h_L + x h_{LG} \longrightarrow x = \dfrac{h - h_L}{h_{LG}}$

Enthalpy along the tube $h = h(0) + \dfrac{2\pi R}{\dot{m}} \displaystyle\int_0^z q(z)\,dz$

Flow regions in convective boiling



The boiling map



Forced convection correlation

Superposition analysis

$$h = h_{nb} + h_{fcv}$$

Dittus-Boelter forced convection correlation

$$h_{fcv} = 0.023\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}_L^{0.4}\,\frac{k_L}{D}$$

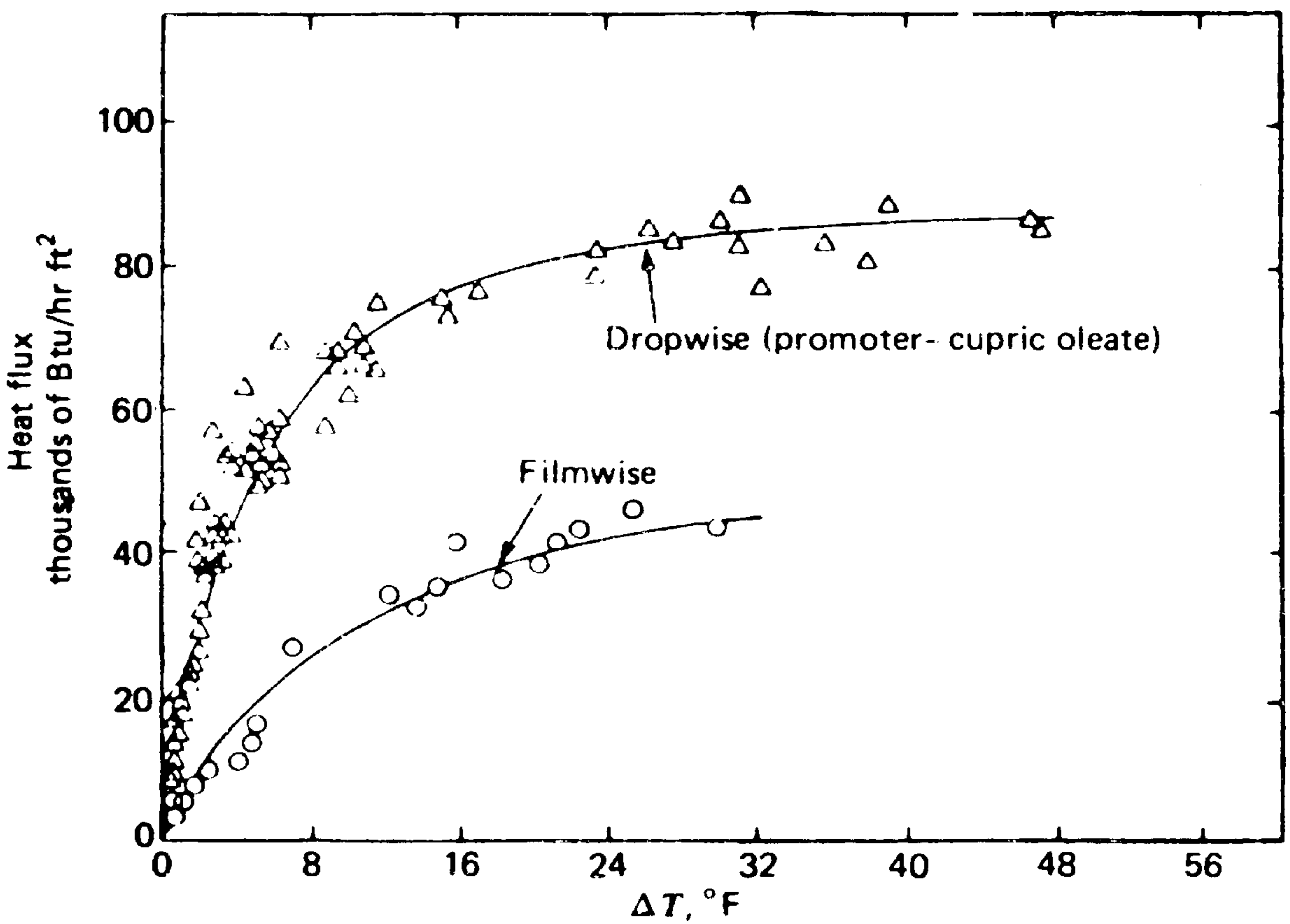
where

$$\mathrm{Re}_L = \frac{G(1-x)D}{\mu_L}$$

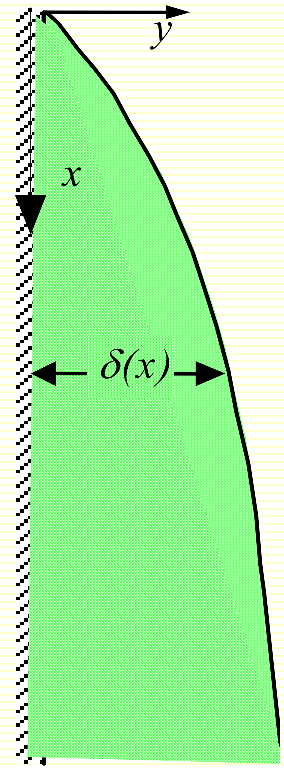
Forster & Zuber (1955)nucleate boiling correlation

$$h_{FZ} = 0.00122\left[\frac{k_L^{0.79}\,c_{pL}^{0.45}\,\rho_L^{0.49}}{\sigma^{0.5}\,\mu_L^{0.29}\,h_{LG}^{0.24}\,\rho_G^{0.24}}\right]\Delta T_{sat}^{0.24}\,\Delta p_{sat}^{0.75}$$

Dropwise vs. film condensation



Condensation on Vertical Surfaces



Continuity equation $$\frac{\partial u_L}{\partial x}+\frac{\partial \upsilon_L}{\partial y}=0$$

Momentum equation $$u_L\frac{\partial u_L}{\partial x}+\upsilon_L\frac{\partial u_L}{\partial y}=\frac{(\rho_L-\rho_G)g}{\rho_L}+\nu_L\frac{\partial^2 u_L}{\partial y^2}$$

Energy equation $$u_L\frac{\partial T_L}{\partial x}+\upsilon_L\frac{\partial T_L}{\partial y}=\sigma\frac{\partial^2 T_L}{\partial y^2}$$

Boundary conditions $$y=0;\qquad u_L=\upsilon_L=0\qquad T=T_w$$

$$y=\delta;\quad \mu\frac{\partial u_L}{\partial y}=\tau_i\qquad T=T_{sat},\qquad k\frac{\partial T}{\partial y}=h_{LG}\frac{d\delta}{dx}$$

Nusselt solution

Heat transfer coefficient

$$h(x)=\frac{q''}{\Delta T}=\left[\frac{k_L^3(\rho_L-\rho_G)gh_{LG}}{4\nu_L x\Delta T}\right]^{1/4}$$

Average heat transfer coefficient

$$\bar{h}=\frac{4}{3}h(L)=0.943\left[\frac{k_L^3(\rho_L-\rho_G)gh_{LG}}{\nu_L L\Delta T}\right]^{1/4}$$

$$\Delta T=T_{sat}-T_w$$

Rohsenow subcooling correction

$$h_{LG}^{©}=h_{LG}\left(1+0.68c_L\Delta T/h_{LG}\right)$$

Variable liquid properties correction

$$T_{ref}=T_w+0.31(T_{sat}-T_w)$$

Turbulent film condensation (Colburn)

$$h=0.074k_L\left[\rho_L(\rho_L-\rho_G)g/\mu_L^2\right]^{1/3}\mathrm{Re}_L^{0.2}\,\mathrm{Pr}_L^{1/2}$$

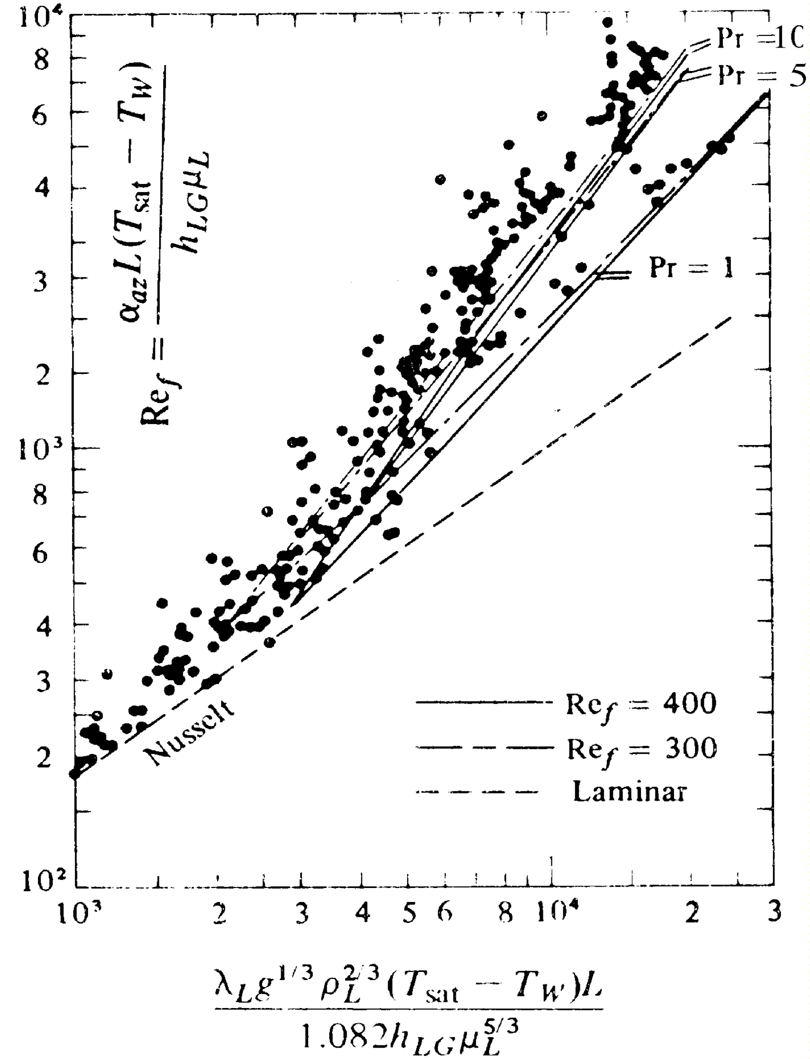
Condensation on Horizontal Tubes

Nusselt analysis

$$\overline{h} = 0.727\left(\frac{(\rho_L - \rho_G) g h_{LG} k_L^3}{D \nu_L \Delta T}\right)^{1/4}$$

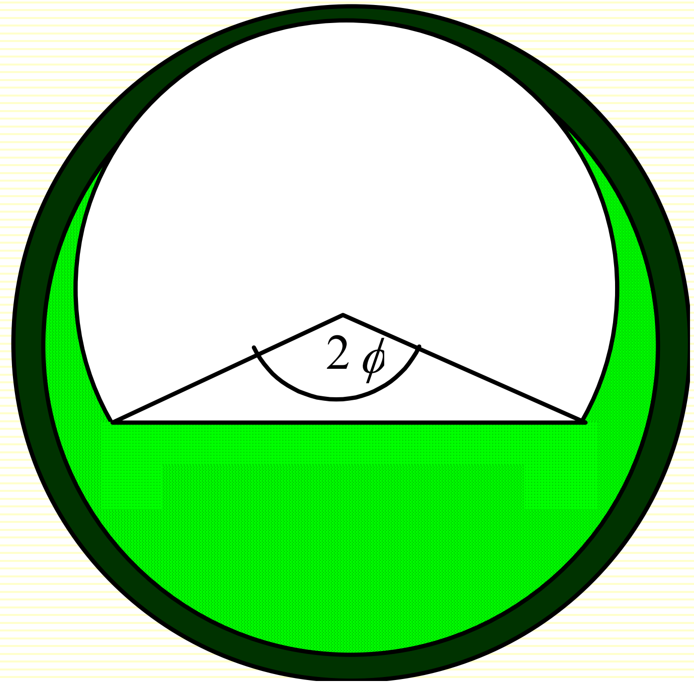
For *n* tubes

$$\overline{h}_n = n^{-1/4}\overline{h}_l$$



Condensation inside horizontal tubes

Chato equation (1962)



$$\bar{h} = 0.557\left[\frac{\rho_L - \rho_G)gh_{LG}k_L^3}{D\nu\Delta T}\right]^{1/4}$$

Boyko and Kruzhilin equation (1967)

$$\frac{\bar{h}D}{k_L} = 0.024\left(\frac{\dot{m}D}{\mu_L}\right)^{0.8}\mathrm{Pr}_L^{0.43}\frac{1+\sqrt{\rho_L/\rho_G}}{2}$$

This equation holds also for inclined and vertical tubes