Bias Error

Occur whenever the FFT spacing is too large

narrow band regions of the "true" PSD. Our spacing Δf is the scale resolution at which we try to resolve the PSD, and some averaging within this spacing occurs for peaks much narrower than this spacing.

An approximation for the relative bias error was developed by Bendat, resulting in

$$e_b = \frac{(\Delta f)^2}{24} S''(f)$$

the bias error , if not negligible, will result in ***underestimation of peaks***

Thus to avoid large bias errors, an analysis bandwidth which is much smaller than 3 db bandwidth is necessary

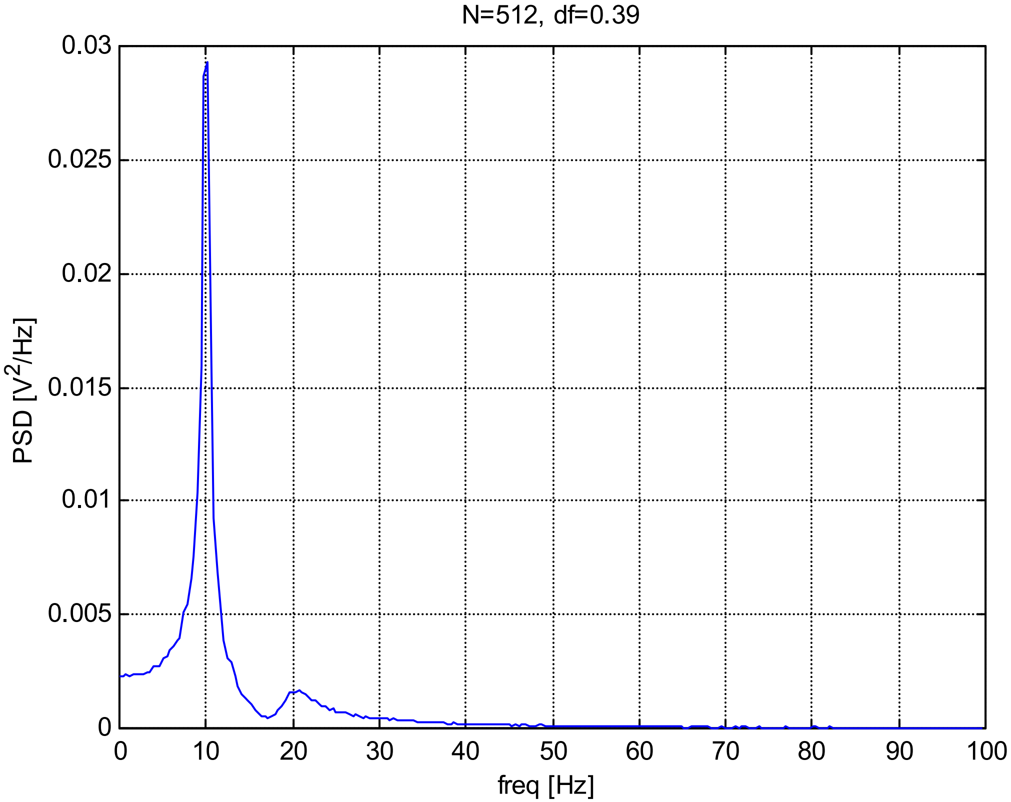
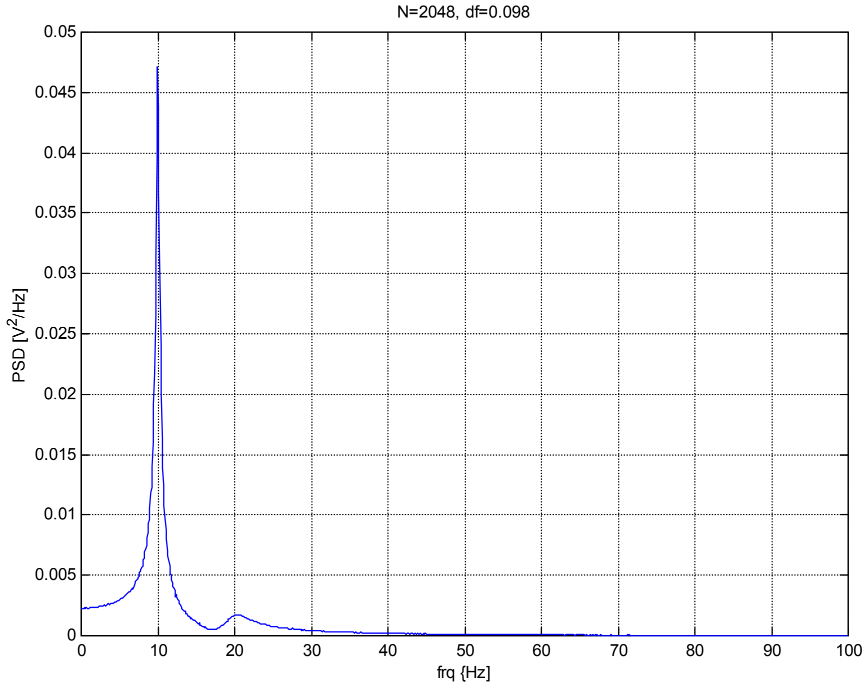

N=512, df=0.39
0.03
0.025
0.02
0.015
0.01
0.005
0
PSD [V^2/Hz]
0
10
20
30
40
50
60
70
80
90
100
freq [Hz]

Figure 4.14a



Spectral Analysis

Random Errors

* We can only compute an estimate of the PSD
* For random data this may have a bias and variance
* The variance is extremely large
* The variance is independent of N

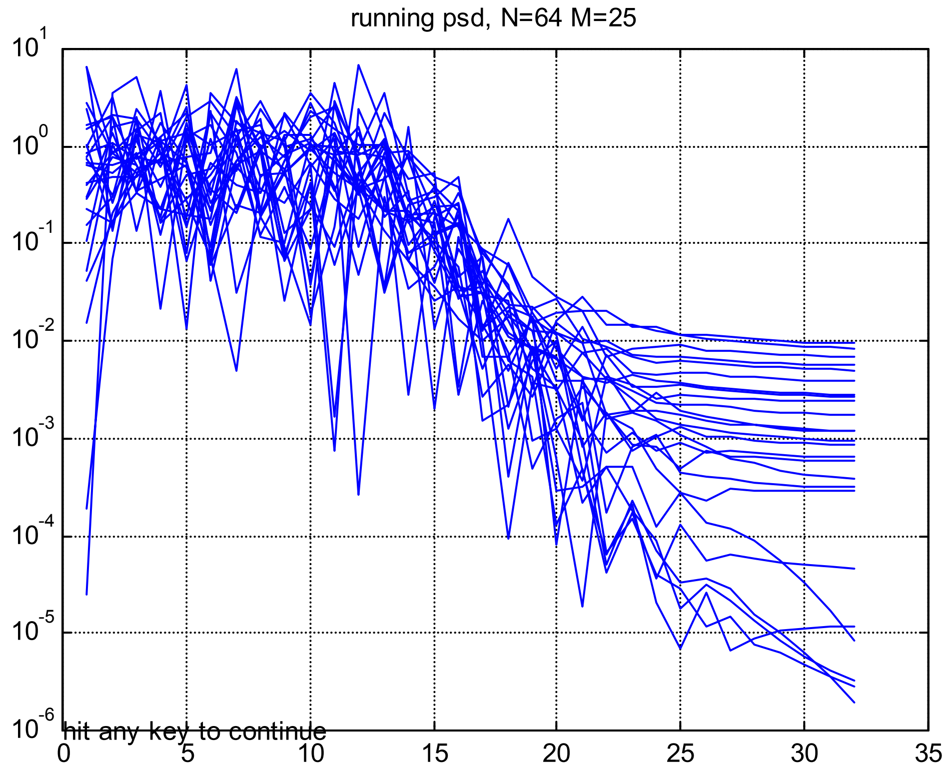


Figure 4.12a

Note that high variability is *independent of N*, the data length

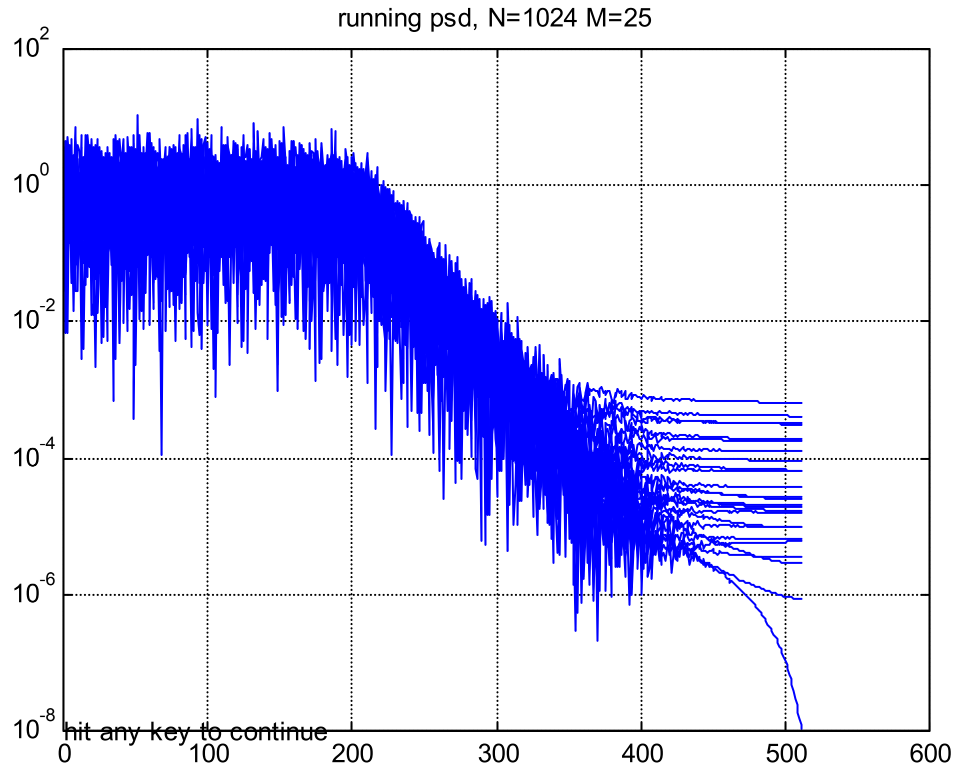

running psd, N=1024 M=25
10^2
10^0
10^-2
10^-4
10^-6
10^-8
hit any key to continue
0
100
200
300
400
500
600

Figure 4.12b

Segment averaging

To reduce the variance we attempt to use averages of estimators of the raw PSD. In the method of segment averaging, estimates to be averaged are obtained from different sections of the signal.

The method is based on sectioning the signal into M non-overlapping segments

$$x_j(i) = x(i) + (j-1)N_1]$$

N1=N/M is the length of each segment, see figure 4.13a

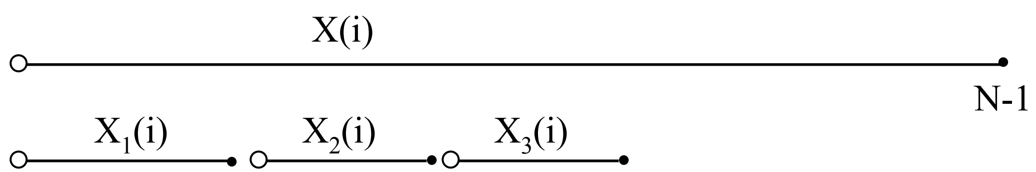


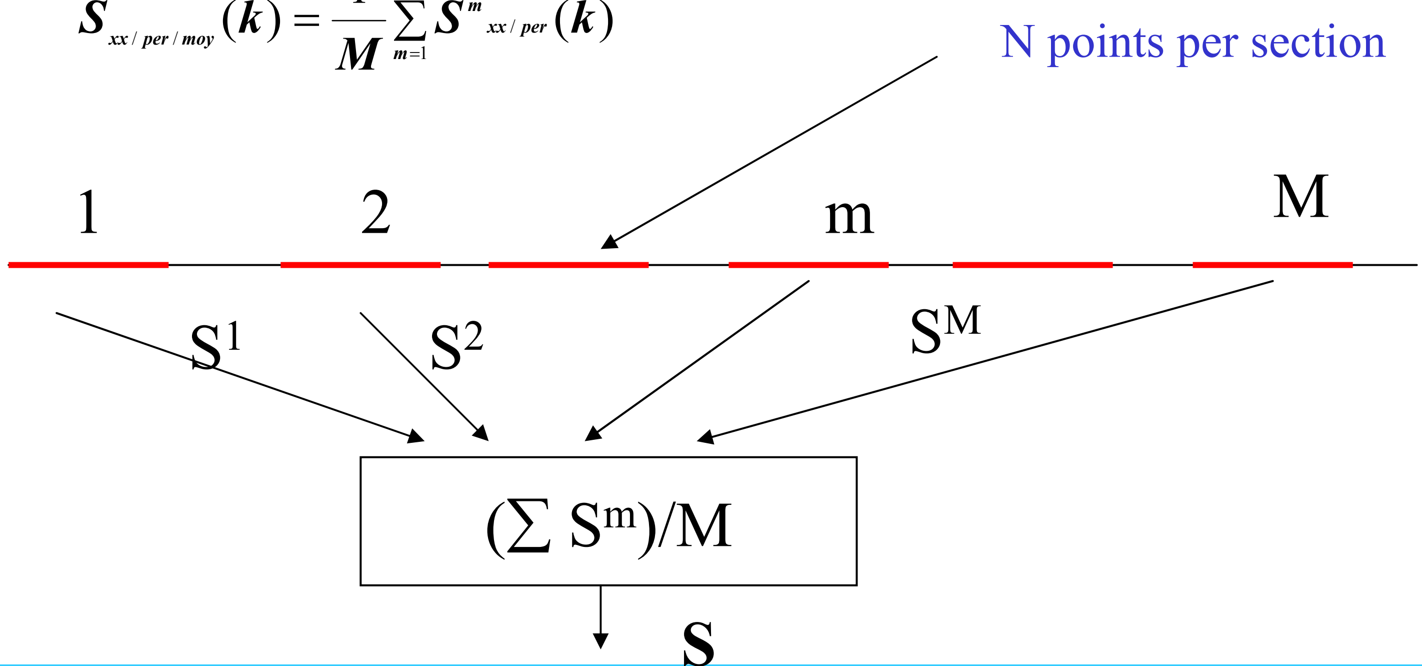
Figure 13a

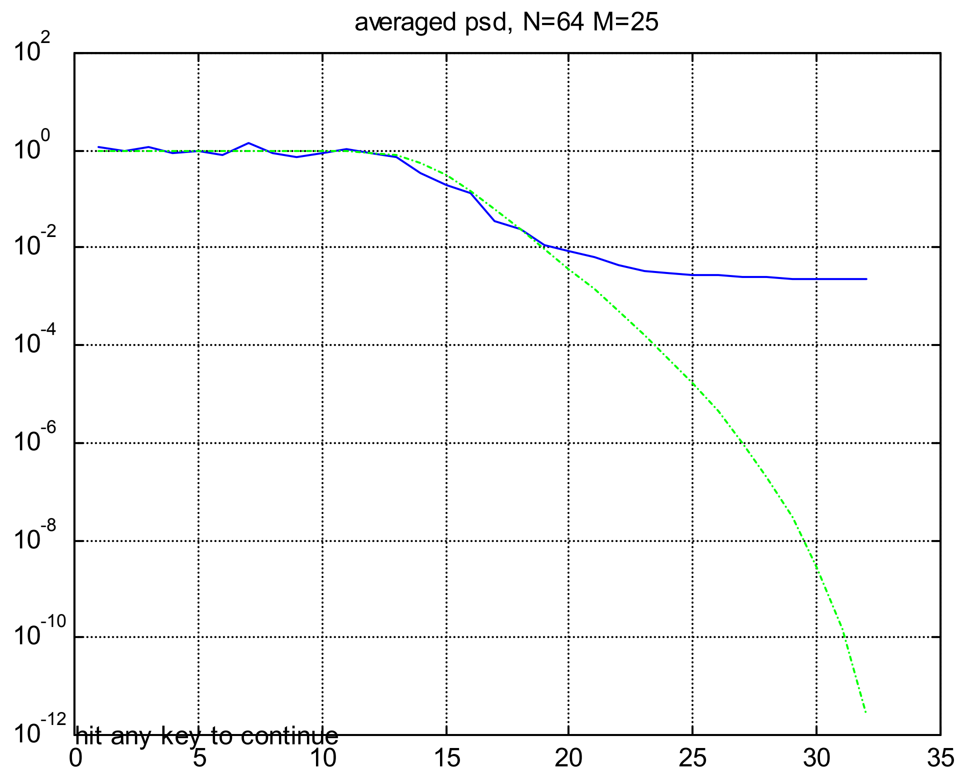
Spectral Analysis

Random error: *Control of Variance*

The basic idea is to average estimators

$$\hat{S}_{xx/per/moy}(k) = \frac{1}{M}\sum_{m=1}^{M}\hat{S}^{m}{}_{xx/per}(k)$$





averaged psd, N=64 M=25
10^2
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
hit any key to continue
0
5
10
15
20
25
30
35

Figure 13b

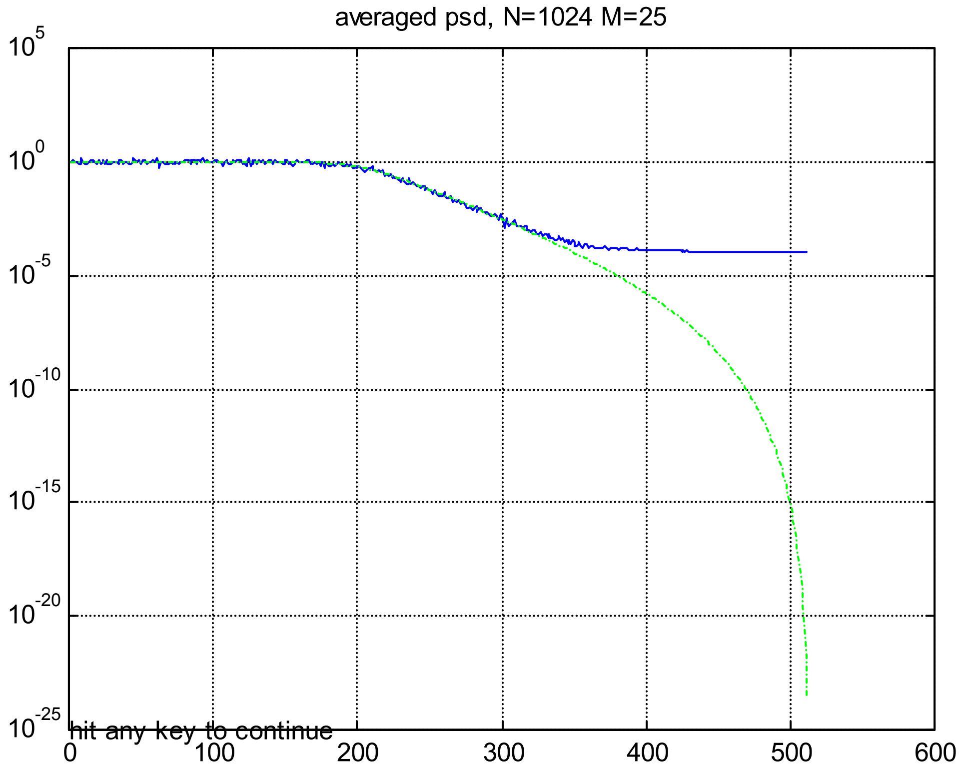

averaged psd, N=1024 M=25
10^5
10^0
10^-5
10^-10
10^-15
10^-20
10^-25
hit any key to continue
0
100
200
300
400
500
600

Figure 13c

The random errors are

$$Average[\hat{S}] = \frac{1}{M}\sum_{i=1}^{M}\hat{S}i \quad \rightarrow S$$

$$Variance[\hat{S}] = \frac{1}{M}\sum_{i=1}^{M}(\hat{S}i - S)^2 \rightarrow \frac{S^2}{M}$$

We thus compute estimates for each segment as

$$\hat{S}_j(k) = \frac{\Delta T}{N_1} \left| X_j(k) \right|^2$$

With

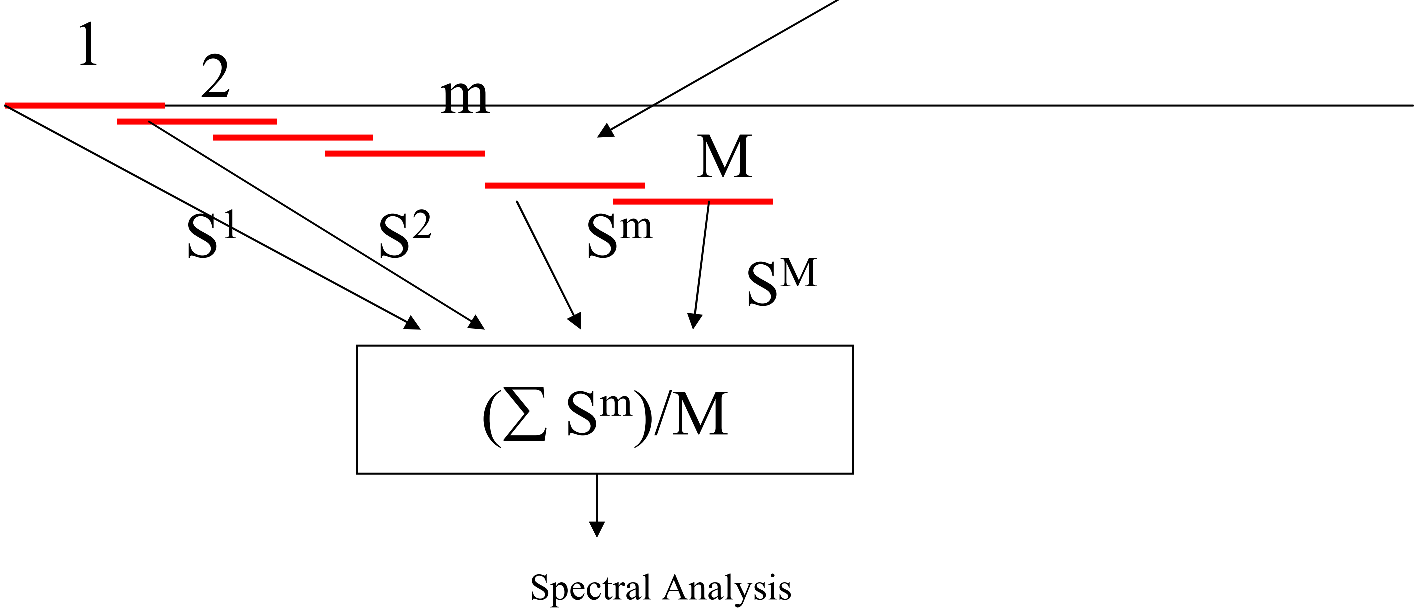
$$x_j(i) \leftrightarrow X_j(k)$$

$$e_r = \frac{1}{[M]^{1/2}}$$

Overlapping

- **We need to increase M to reduce variance**
  - **The analysis time T_{max} >N.M.ΔT could be too large**

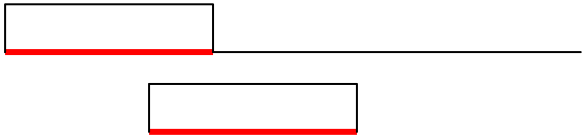
$$\hat{S}_{xx/per/moy}(k) = \frac{1}{M}\sum_{m=1}^{M}\hat{S}^{m}{}_{xx/per}(k)$$



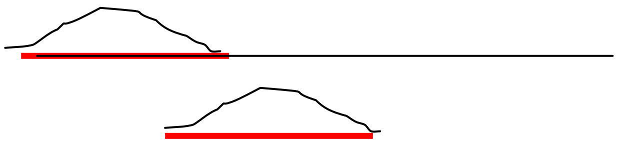
Overlapping(2)

- **Method to decrease total time needed**
- ***But sections are not ‘ independent’ anymore:***

  - Variance decreses more slowly with N
  - The windows make sections independent



Rectangular window



Hanning window

Control of errors (1)

The data is unlimited, and any required number of samples can be acquired. Such would be the case for rotating machinery, where the limitation would probably be the existence of a stationary vibratory regime

Specifications for analysis would be

Bandwidth of analysis – setting Δt during acquisition

Frequency resolution – setting N the block length

Random error – setting M, the number of averages

As an example, assume a required bandwidth of 100 Hz, analysis resolution of $\Delta f_{res} = 0.5$ Hz

and a random error of 10%

Control of errors (1) −*contnd*:

Assuming the existence of a sampling interval of (the exact values will depend on the actual hardware used)

$\Delta t=1/(2.5\ f_{max})=1/250=4$msec

The block length would be

$1/(\Delta t\ \Delta f_{res})=1/(0.04*0.5)=500$

and we would choose N=512

For a random error of 10% (eq 4.29)

We would choose M=100

And the total number of samples is NM=512*100= 51200, corresponding to a data duration of

$Tt=NM\Delta t=204.8$ sec

Control of errors (2)

-

Assume data was acquired for 60 seconds , ie number of data samples is 60/Δt =15000. As now M*N is constant, we can only control one error, the bias via N, or the random one via M.

Case 1: Random error fixed, M=100, N=150. Thus we may choose

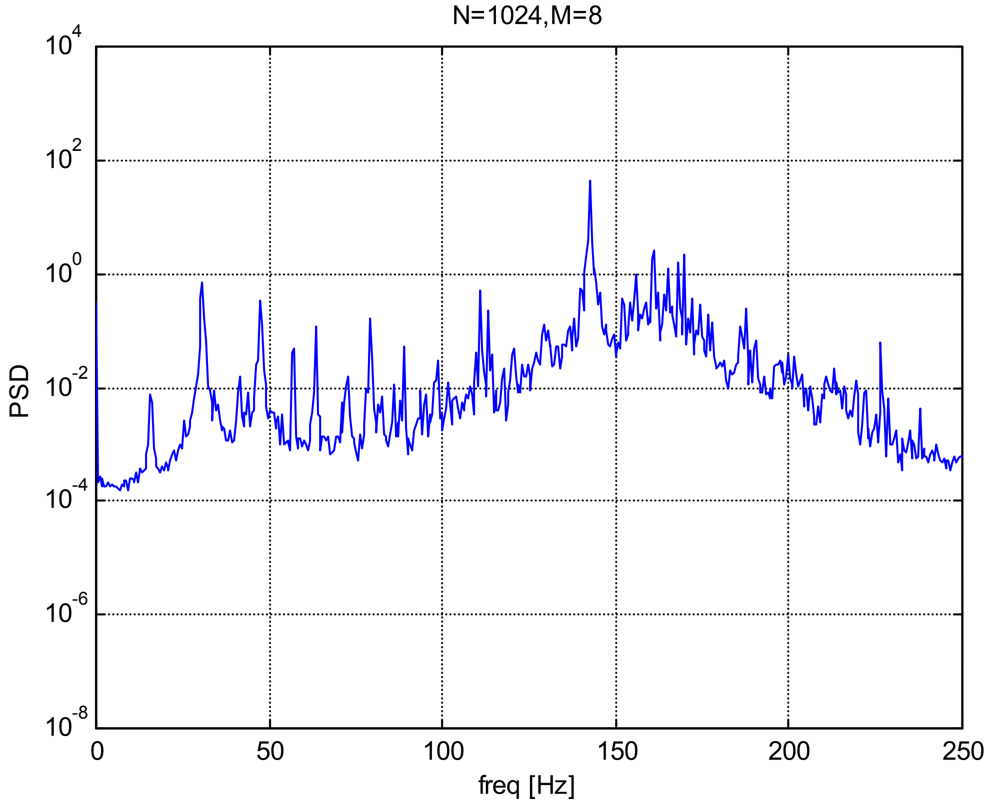
N1=128 with a resultant resolution $\Delta f = 1.95$ Hz (and the actual M=117, $\varepsilon_r = 9.3$ %)

N2=256 with the resultant resolution of $\Delta f = 0.95$ Hz (and the actual M=58, $\varepsilon_r = 13.1\%$)

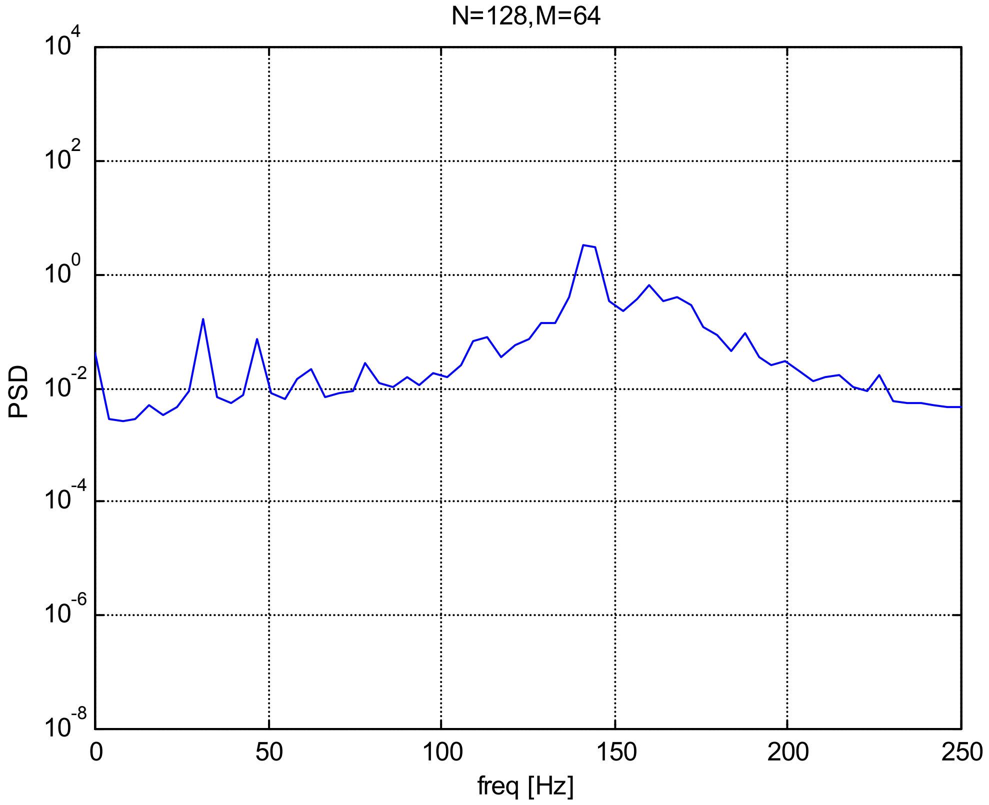
Case 2: Resolution fixed as in scenario 1: N=512

With M=15000/512=29 and $\varepsilon_r = 18.6\%$

Thus for a-priori fixed data length, the random error can only be balanced by the bias error.

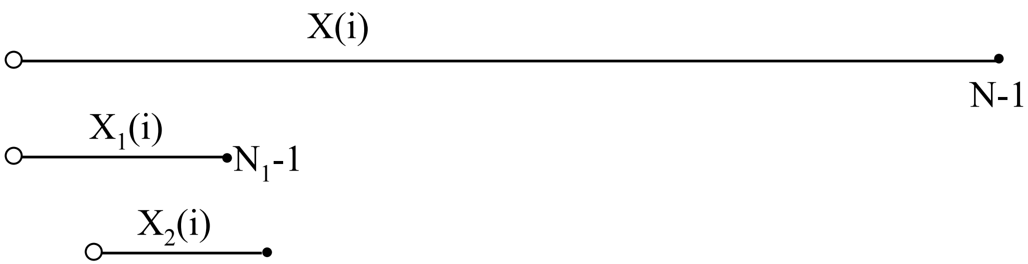


Spectral Analysis


N=128,M=64
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
PSD
0
50
100
150
200
250
freq [Hz]

	Computation	EU	Voltage Units
FT	$\Delta T X$	V-sec	
FS	X/N	V	
S(PSD)	$\frac{\Delta T}{N}\lvert X\rvert^2$	$V^2/Hz=$ $=V^2$-sec	$V/\sqrt{Hz}$
S_E(ESD)	$\Delta T^2\lvert X\rvert^2$	V^2-sec^2	V-sec
Power in bin	$S_E\Delta f = \frac{\Delta T}{N}\lvert X\rvert^2$	V^2	
Energy in bin	$S_E\Delta f = \frac{\Delta T}{N}\lvert X\rvert^2$	V^2-sec	
Total Power (frequency)	$P_f = \frac{1}{N^2}\sum\lvert X\rvert^2$	V^2	
Total Power (time)	$P_t = \frac{1}{N}\sum x^2$	V^2	
Total Energy (frequency)	$E_f = \Delta T\sum X^2$	V^2-sec	
Total Energy (time)	$E_t = \Delta T\sum x^2$	V^2-sec	

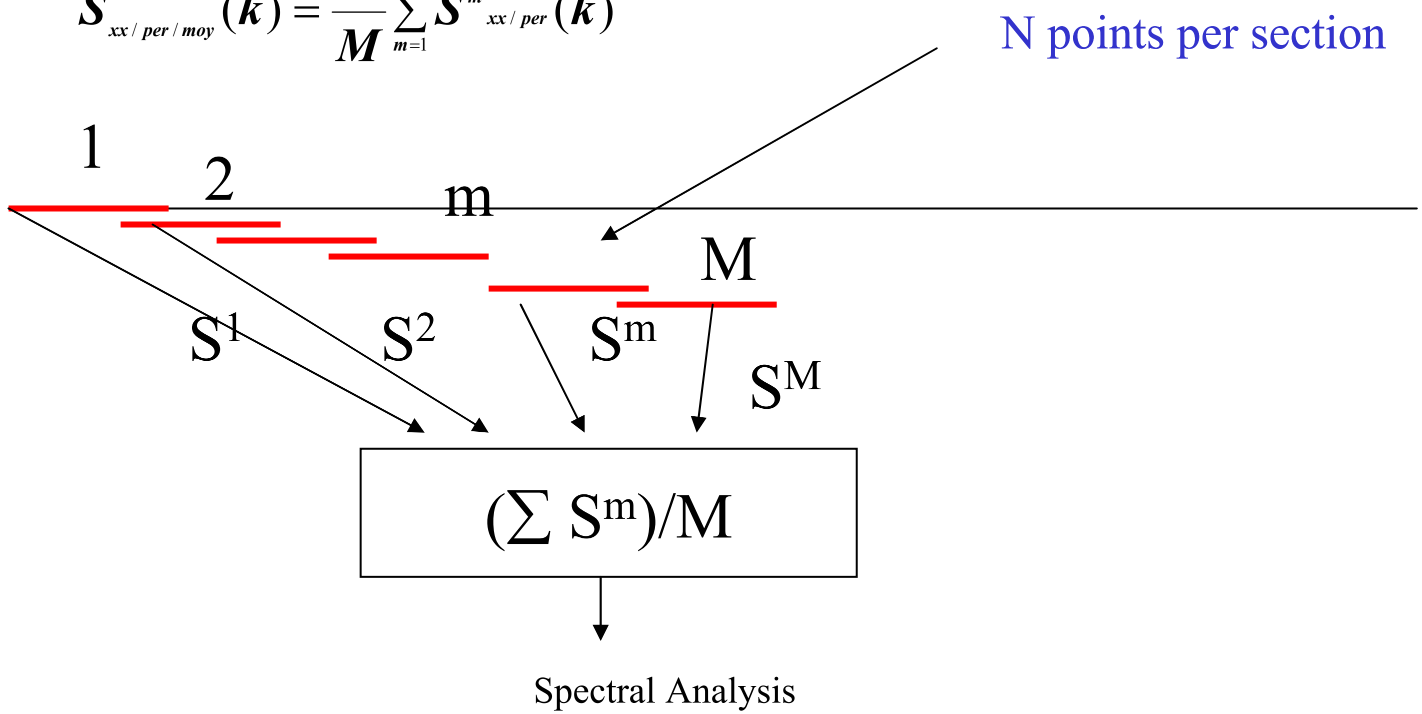
$$Parseval: \sum x^2 = \frac{1}{N}\sum|X|^2$$


X(i)
N-1
X1(i)
N1-1
X2(i)

Overlapping

- **We need to increase M to reduce variance**
  - **The analysis time T_{max} >N.M.ΔT could be too large**

$$\hat{S}_{xx/per/moy}(k) = \frac{1}{M}\sum_{m=1}^{M}\hat{S}^{m}{}_{xx/per}(k)$$

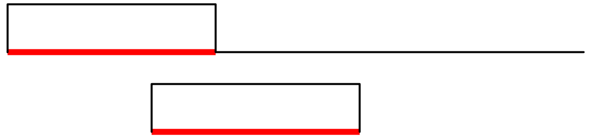


To retain the statistical independence of the raw PSDs, the segments are windowed, so as to give less weights to the overlapped points. The analysis of Welch suggest an overlap of 50% in conjunction with a Hanning window , and the PSD of each section is thus modified to

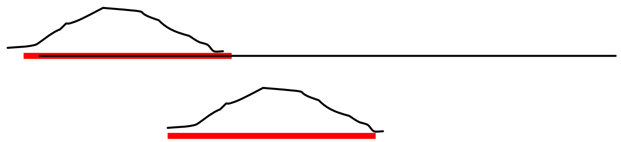
$$\hat{S}_{j(k)} = \frac{\Delta T}{N_1} \left| X_j(k) W(k) \right|^2$$

Overlapping(2)

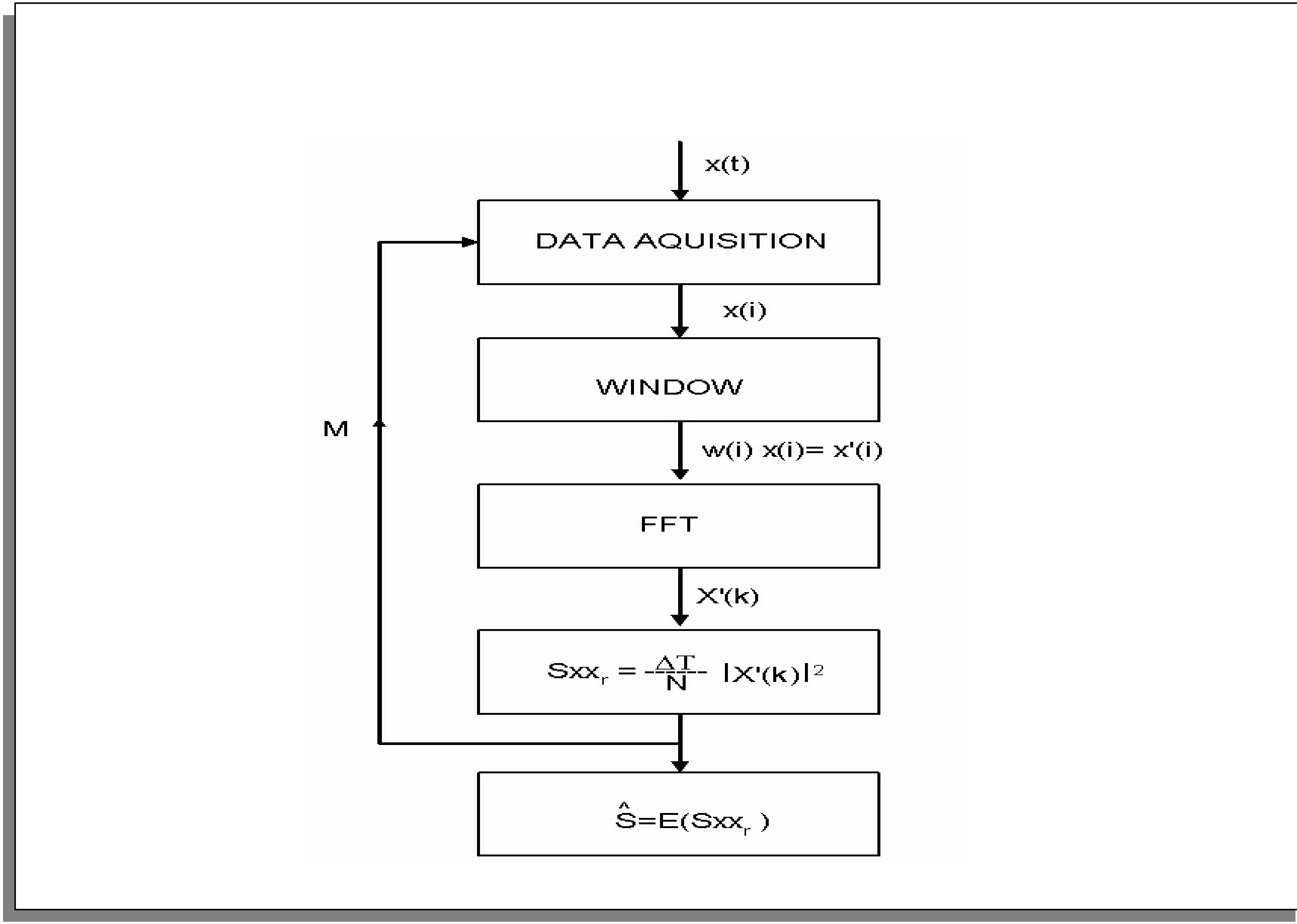
- **Method to decrease total time needed**
- ***But sections are not ' independent' anymore:***
  - Variance decreses more slowly with N
  - The windows make sections independent



Rectangular window



Hanning window



Spectral Analysis

Spectral Analysis

Spectral Analysis