Spectral Analysis

Rationale

* Physical insight
* Orthogonality (no crossproducts)
* Pattern recognition
* Algebraic (closed form) solutions, as with many transform based methods
* Performance, Standards

Frequency domain presentation

Depends on signal class

* Transient

* Periodic

* Random

All computations via basic FFT

For a signal

$$x(i\Delta t) \qquad i=0,1\ldots\ldots N$$

We have computed the FFT

$$X(k\Delta f) = \sum_{i=0}^{N-1} x(i)\exp(-j\frac{2\pi}{N})^{ik} \qquad k=0,1\ldots\ldots N-1$$

Frequency scale

$$\Delta f = 1/(N\Delta t)$$

$$f(k) = k\Delta f \qquad k=0,1\ldots\ldots N-1$$

Transients:

The DFT approximates samples of the continuous FT.

For EU

$$X_{EU}(k\Delta f) = \Delta T\ X(k\Delta f) \quad [V - \sec]$$

Periodic signals:

$$X_{DFS}(f_k) = \frac{1}{N} X(k) \qquad [V]$$

This is a 2 sided representationThe computation equals that of the DFT except for the factor of N. The results are directly in [Volts]For a one sided representation

$$X_{DFS}(f_k) = \frac{1}{N/2} X(k) \qquad [V]$$

$$For \quad k = 1 \ldots (N/2 - 1)$$

$$X_{DFS}(f_k) = \frac{1}{N} X(k) \qquad [V]$$

$$For \quad k = 0, k = N/2$$

Assume that p integer periods are spanned by the signal length $N\Delta t$. Denoting the number of samples in the period by M

$$T_p = M\Delta t$$

with T_p the actual period of the physical signal. The total signal length

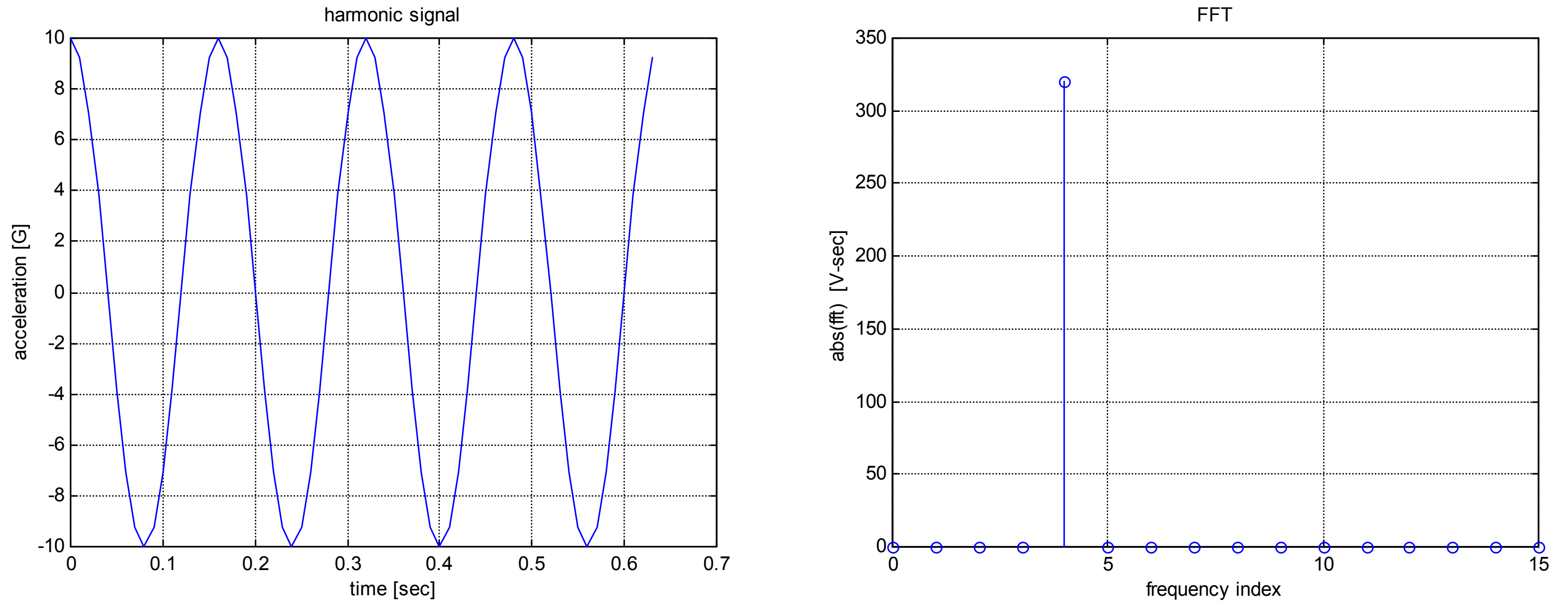
$$N\Delta t = pM\Delta t$$

and the location of f_p on the frequency scale is

$$k\Delta f = f_p = 1/(M/\Delta t)$$

$$k=1/(M\ \Delta t\ \Delta f)=N/M=p$$

when an integral number of periods are spanned , the physical frequency coincide with one of the frequencies at which the DFS is computed.

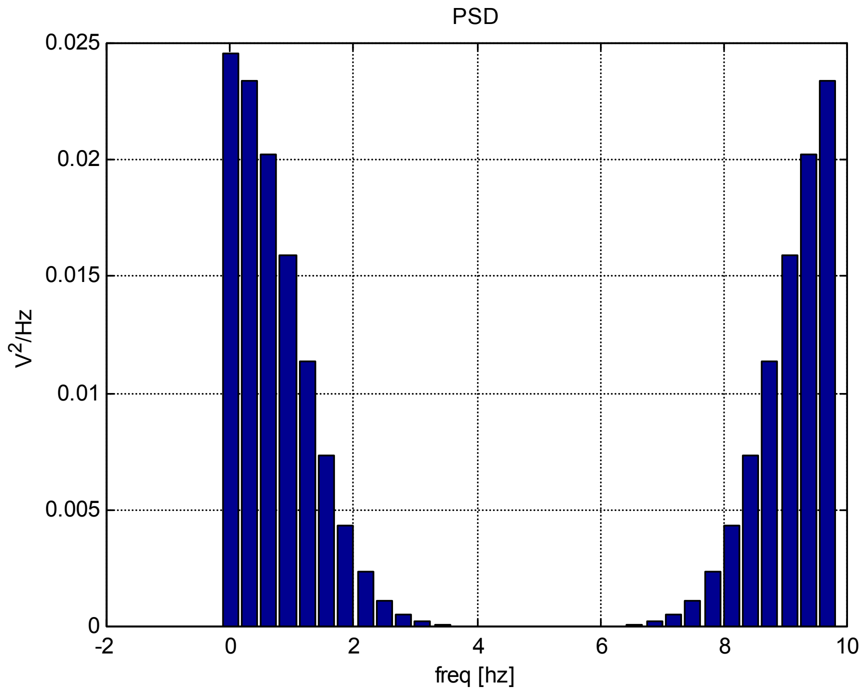


Signal Processing: Spectral Analysis

Random continuous signals:

$$P = \int_{-\infty}^{\infty} S(f)\,df$$

$$P_{f_1 - f_2} = \int_{f_1}^{f_2} S(f)\,df \qquad S(k\Delta f) = \frac{\Delta T}{N}\left|X(k\Delta f)\right|^2$$

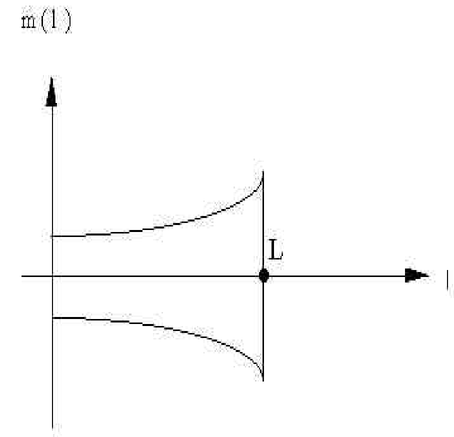


Distribution Function

$$Q = \int_{\alpha_1}^{\alpha_2} q(\alpha)d\alpha$$

Example - mass distribution

$$M = \int_{\ell_1}^{\ell_2} m(b)d\ell$$



Example   Power Distribution in the frequency domain

PSD = Power Spectral Density

$$P = \int_{f_1}^{f_2} S(f)df$$

Spectral Analysis
Engineering Units

- x( ) en Volts

	Power	PSD	Energy	ESD
•	V^2	V^2/Hz	$V^2.sec$	$V^2.sec/Hz=V^2.sec^2$

- Voltage units

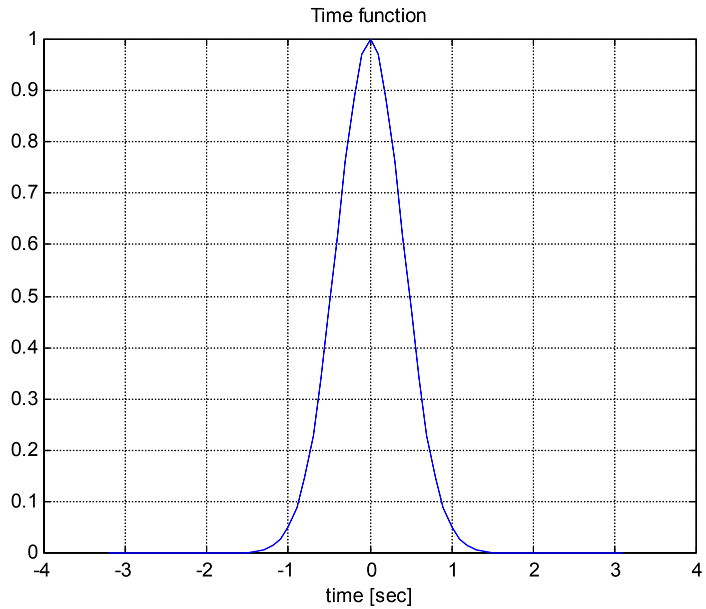
•	V	$V/\sqrt{Hz}$	$V.\sqrt{sec}$	V.sec

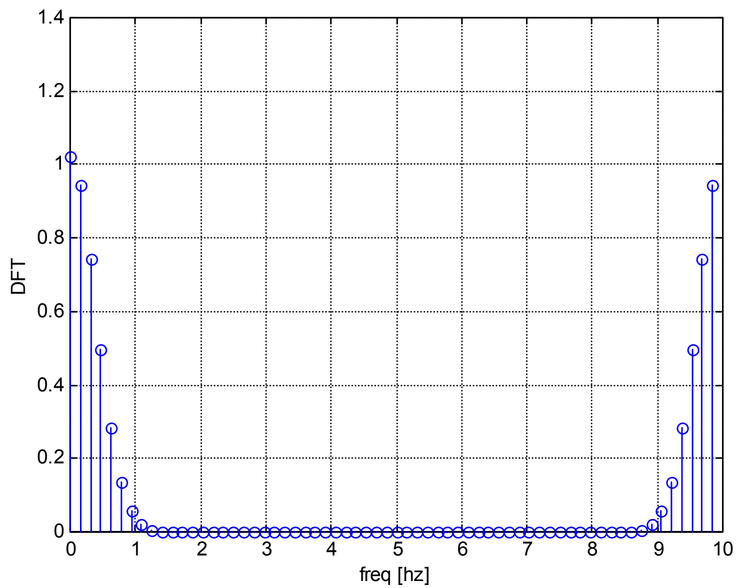
Frequency scales, one and two sided presentations :

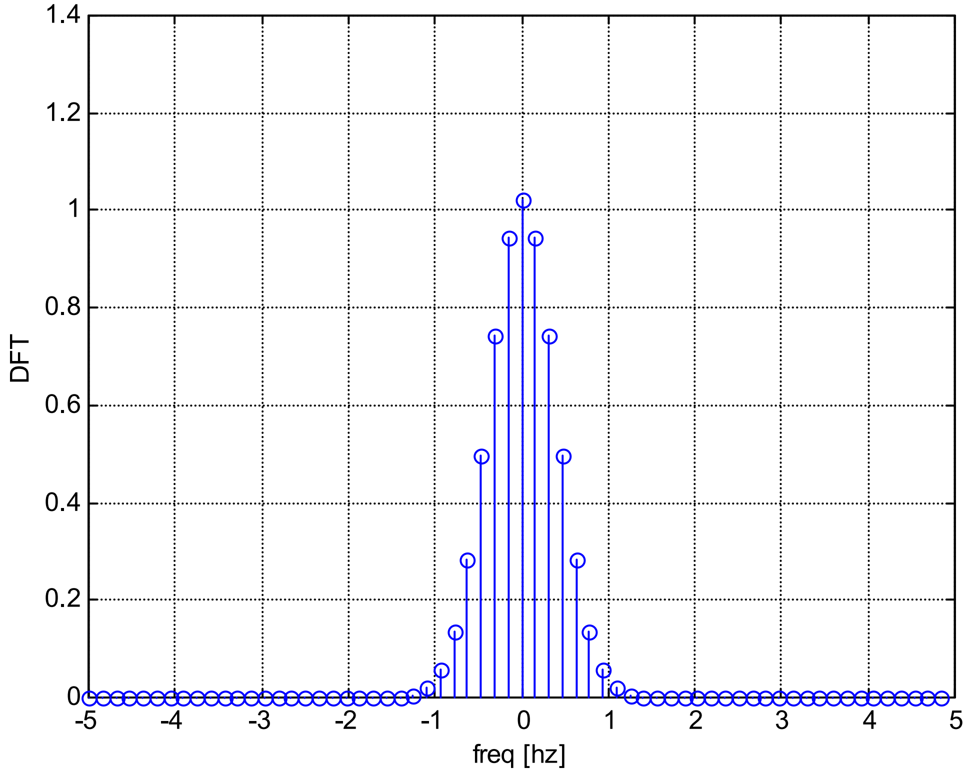
The (computational) resolution is Δf=1/(N/Δt) , the frequency scale is f(k) = kΔf

the locations k = N-1,N,…….N/2

correspond to negative frequencies according to: X(-k)=X(N-k)

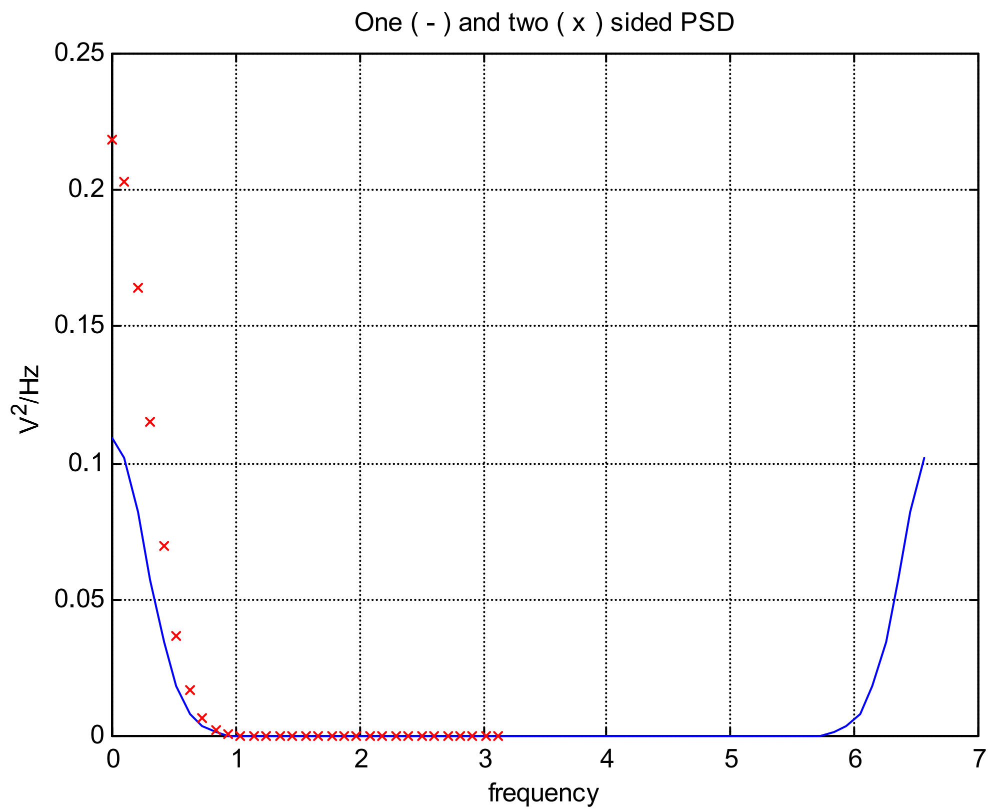


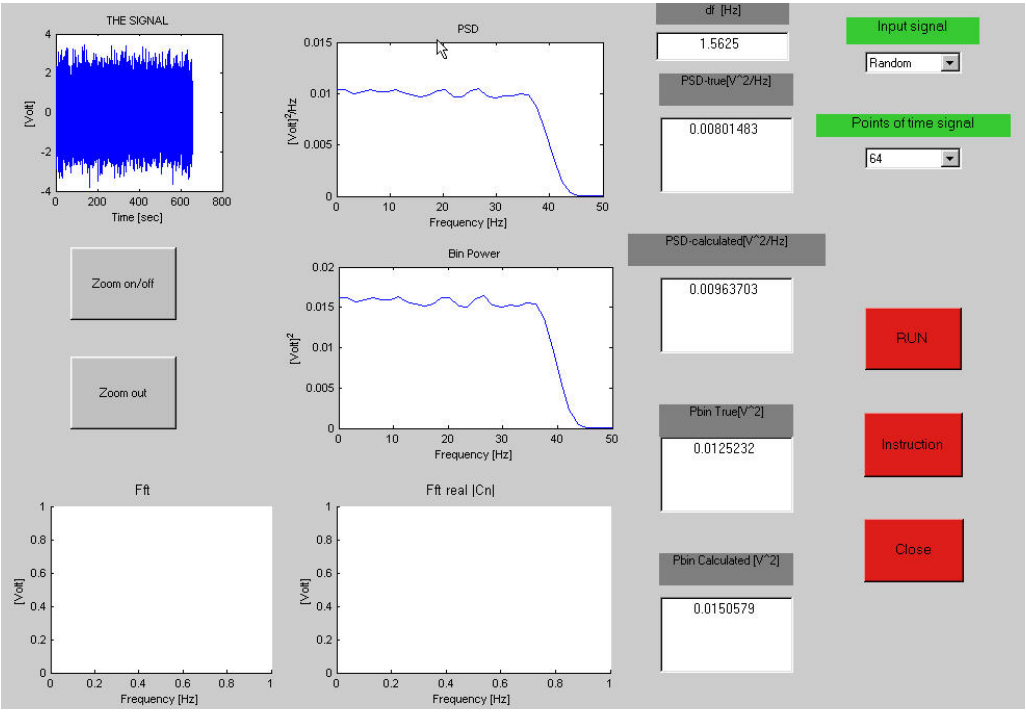



1.4
1.2
1
0.8
0.6
0.4
0.2
0
DFT
-5
-4
-3
-2
-1
0
1
2
3
4
5
freq [hz]

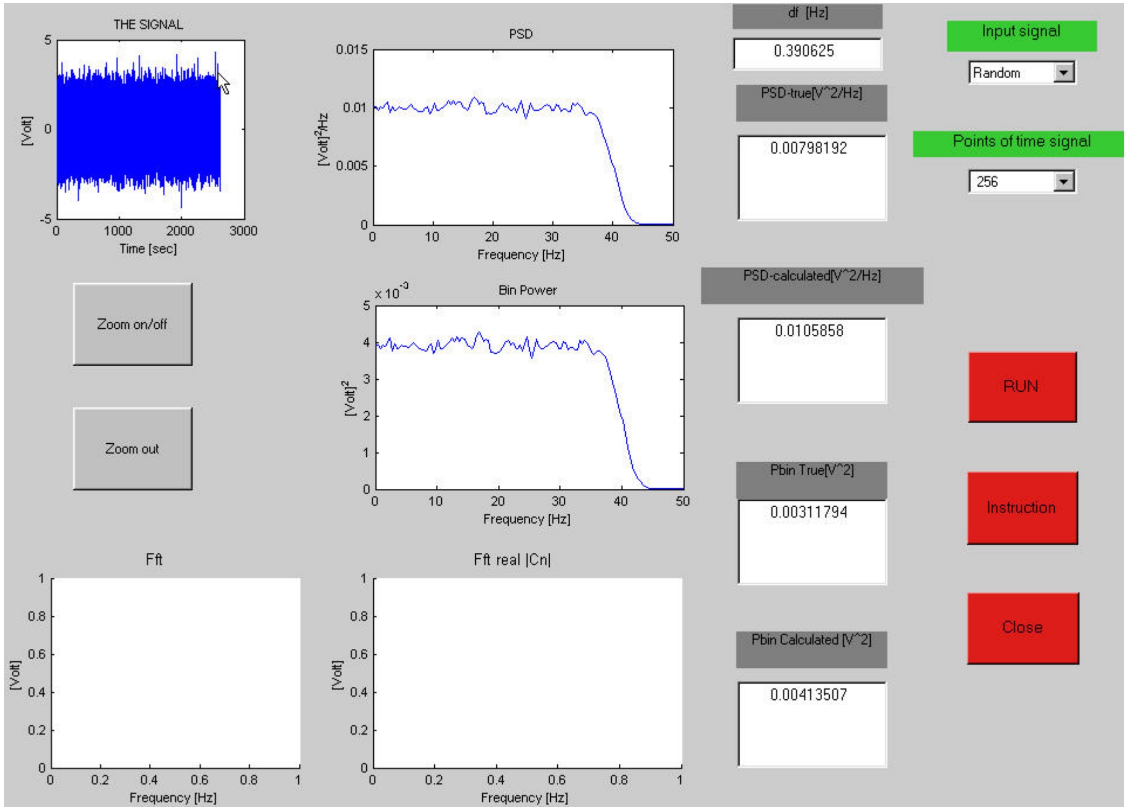
$$X_1(k) = \begin{cases} X(k) & k = 0,\ k = N/2 \\ 2X(k) & k = 1, 2 \ldots N/2 - 1 \end{cases}$$

$$G(k) = \begin{cases} S(k) & k = 0,\ k = N/2 \\ 2S(k) & k = 1, 2 \ldots \frac{N}{2} - 1 \end{cases}$$





Signal Processing: Spectral Analysis



Signal Processing: Spectral Analysis

The uncertainty principle :

For any reasonable definition

(Time duration) x (Frequency bandwidth) > C

given a signal length T_t, two components separated by

$f_2 - f_1 = 1/T_t$ will not be separated by *any* signal

processing techniques.

Transient analysis and zero padding

The frequency resolution is $\Delta f_{transient} = \dfrac{1}{n\Delta T}$

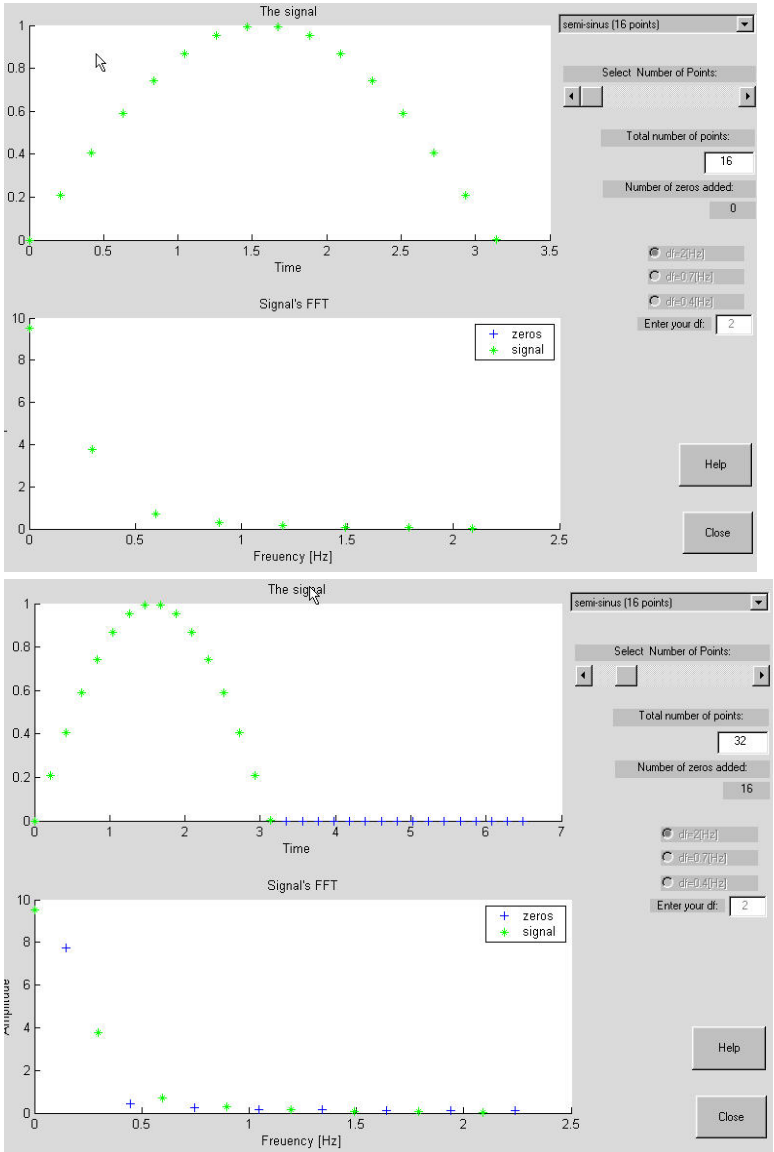
N-n zeros are now appended to the signal, resulting in total span of N points. The computational resolution is

$$\Delta f = \frac{1}{N\Delta T} < \Delta f_{transient}$$

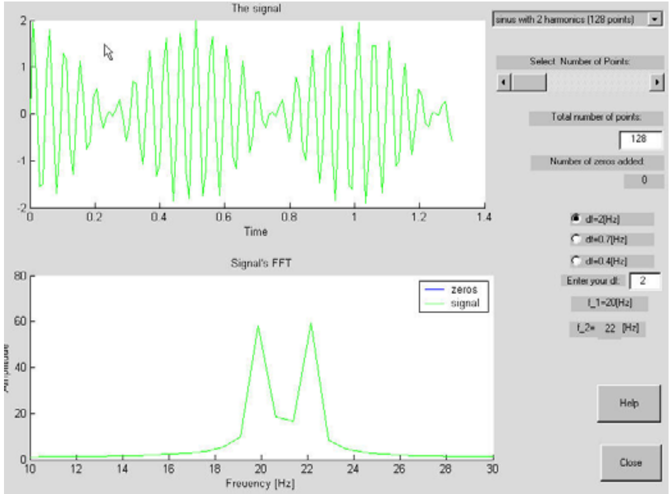
The DFT

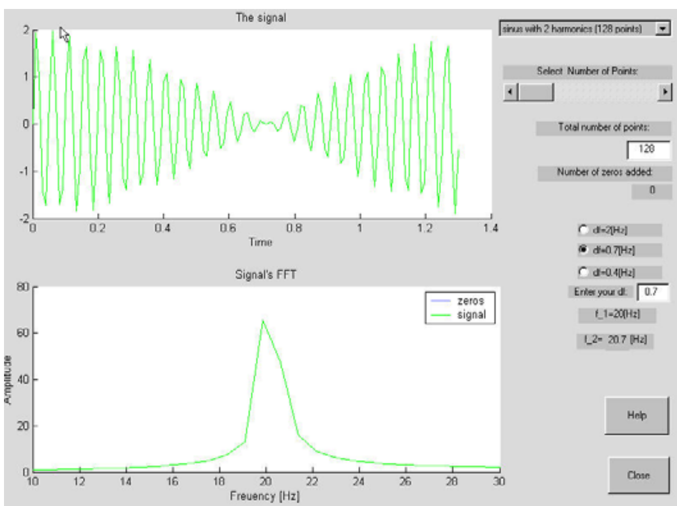
$$X(k) = \sum_{i=0}^{n-1} x(i) \exp(-j\frac{2\pi}{N})^{ik} + \sum_{i=n}^{N} 0 \ \exp(-j\frac{2\pi}{N})^{ik}$$

Nothing is contributed to X(k) by the second term.

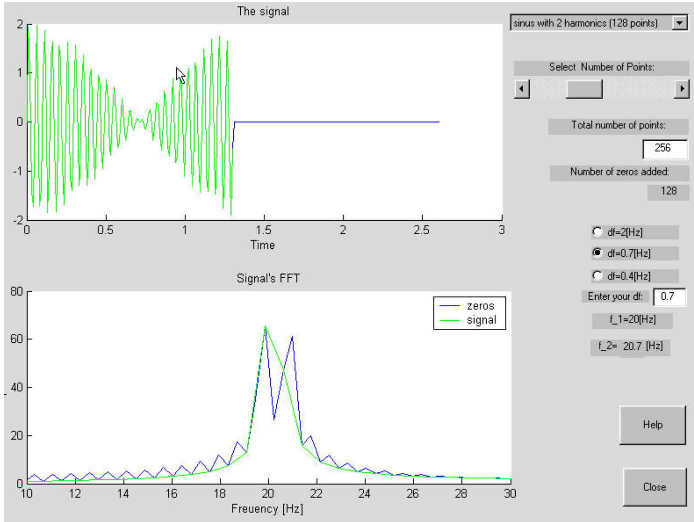


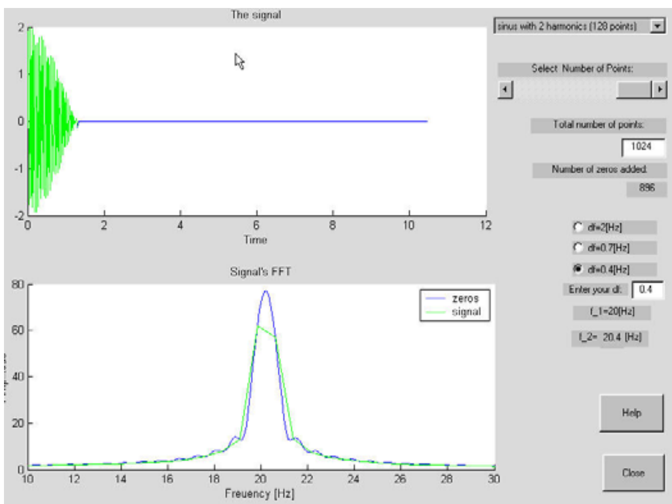
Signal Processing: Spectral Analysis





Signal Processing: Spectral Analysis


The signal
sinus with 2 harmonics (128 points)
Select Number of Points:
Total number of points:
256
Number of zeros added:
128
df=2[Hz]
df=0.7[Hz]
df=0.4[Hz]
Enter your df:
0.7
f_1=20[Hz]
f_2= 20.7 [Hz]
Help
Close
Time
Signal's FFT
zeros
signal
Freuency [Hz]


The signal
sinus with 2 harmonics (128 points)
Select Number of Points:
Total number of points:
1024
Number of zeros added:
896
df=2[Hz]
df=0.7[Hz]
df=0.4[Hz]
Enter your df:
0.4
f_1=20[Hz]
f_2= 20.4 [Hz]
Help
Close
Time
Signal's FFT
zeros
signal
Freuency [Hz]

Performance, errors and controls:

The main error mechanisms in spectral analysis

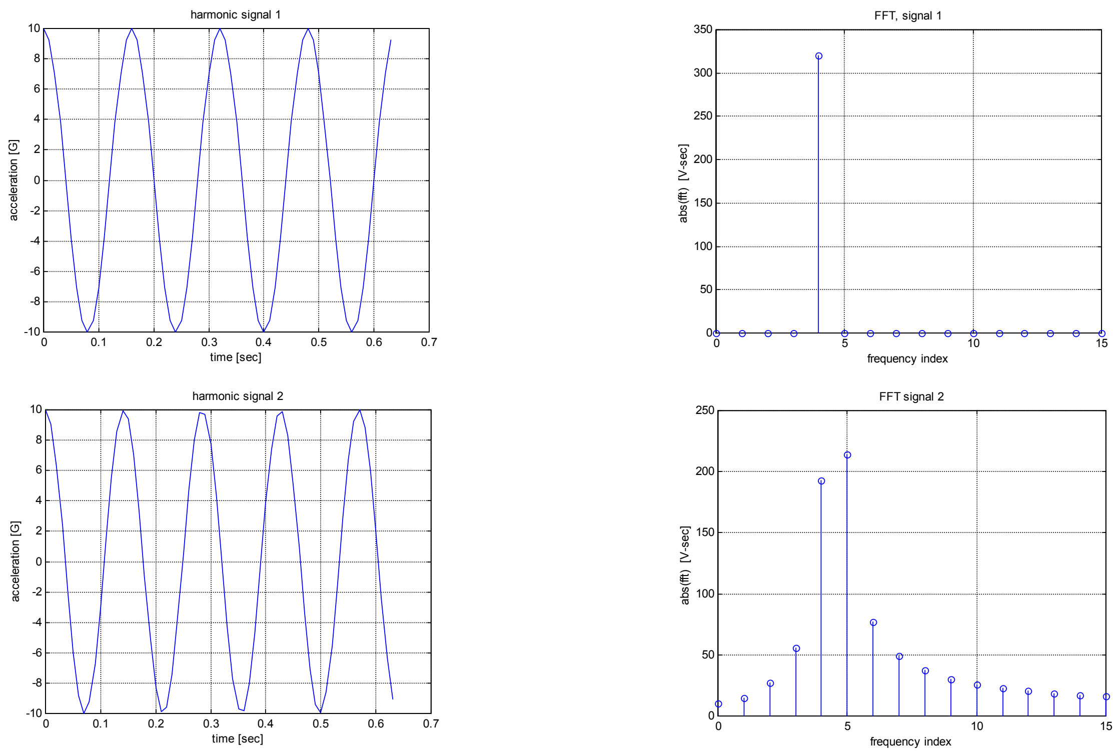
- Alias errors
- Leakage errors
- Random errors
- Bias errors

Periodic signals -Leakage

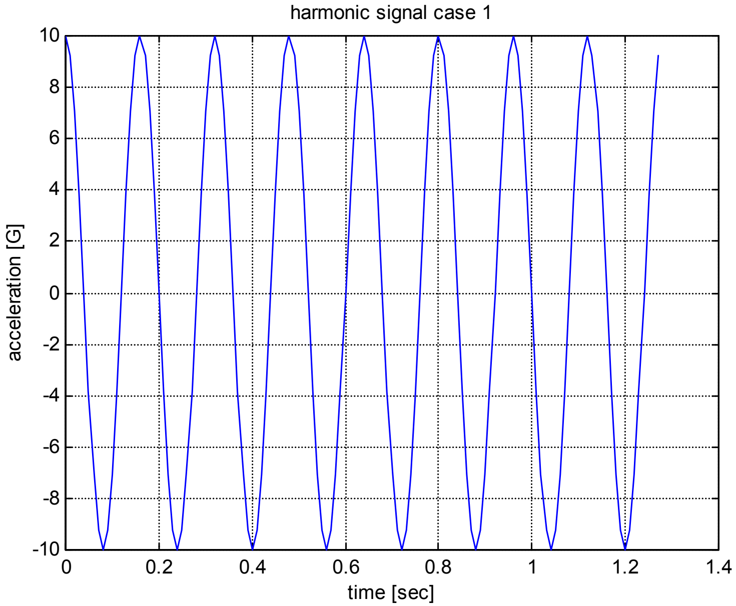
For periodic signals , with period T_p, the physical frequencies are

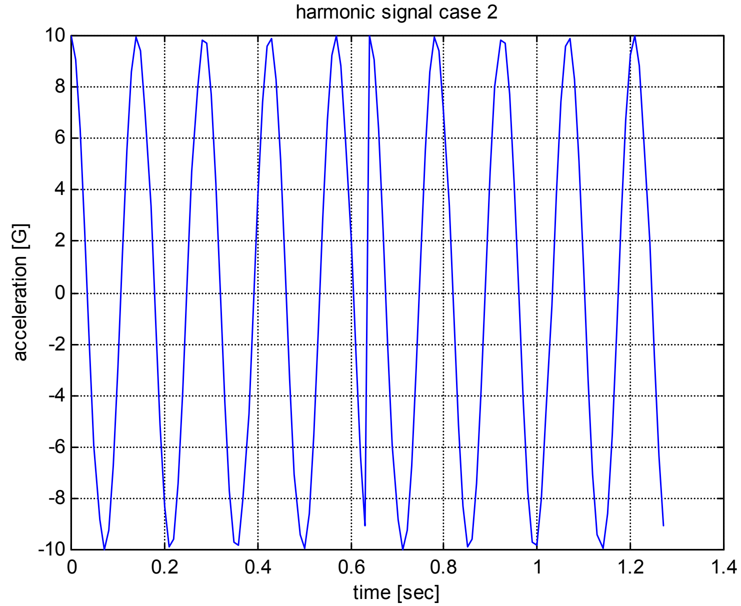
$$k/T_p, \text{ with } k=1,2\ldots$$

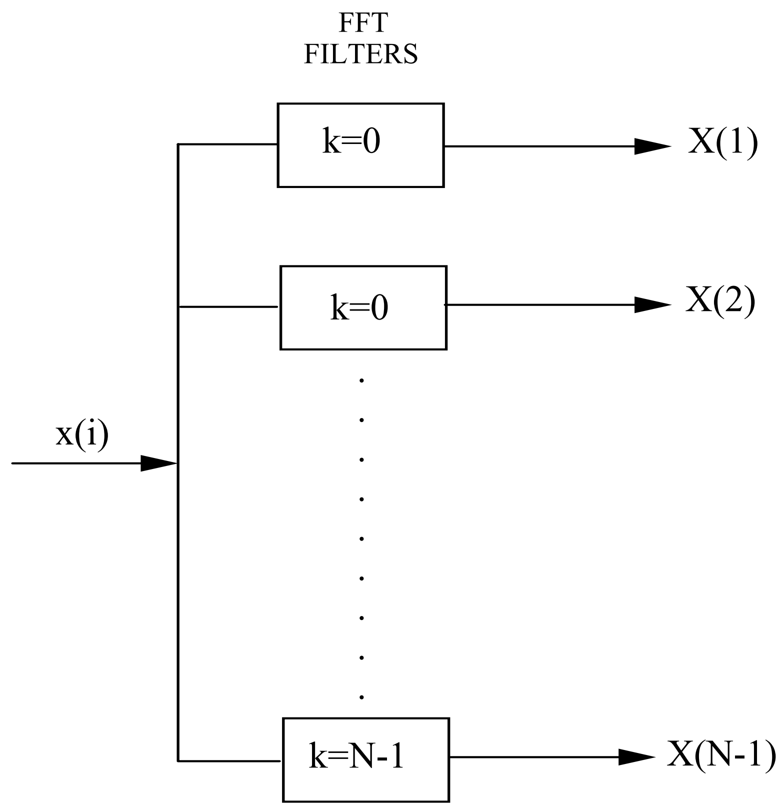
The computed frequencies will be located at $k\Delta f = k/(N\Delta t)$



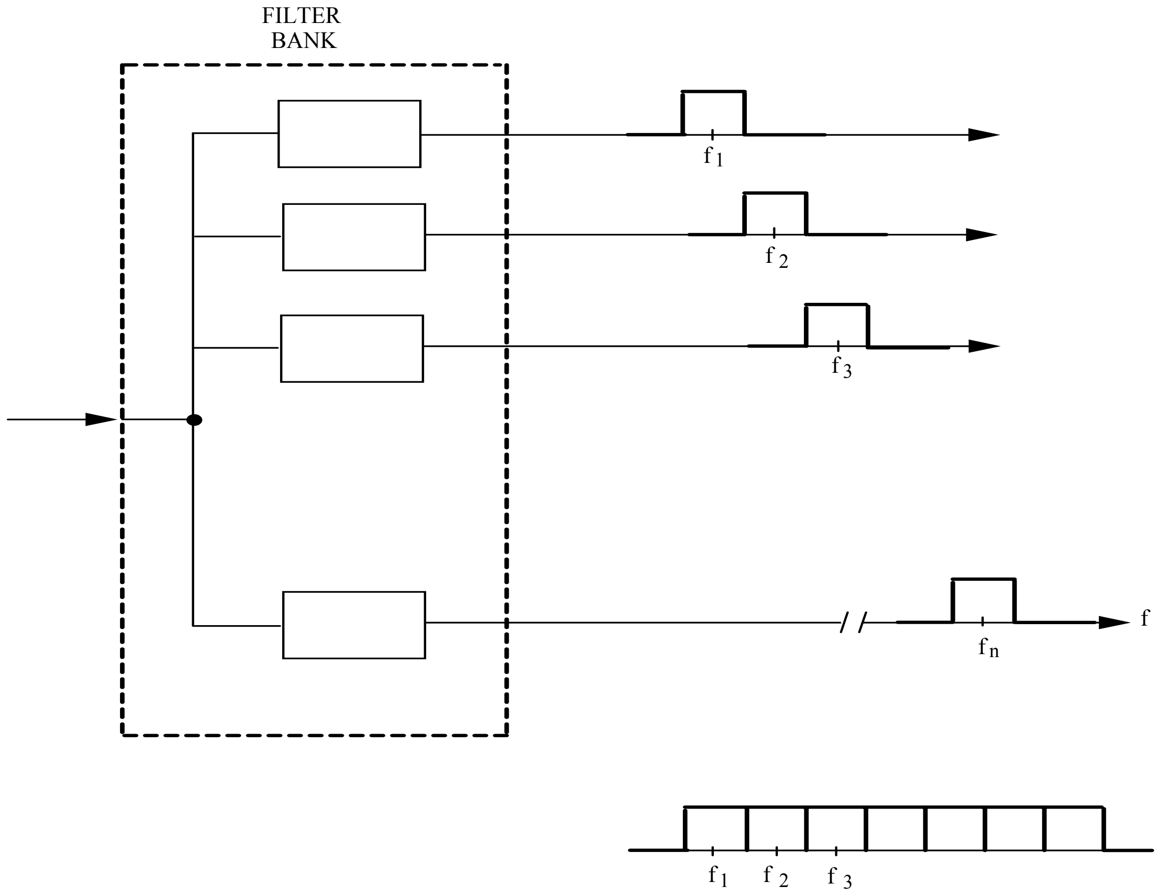
No discontinuities exist when extending periodically an harmonic (or any periodic) signal composed of an integer number of periods.



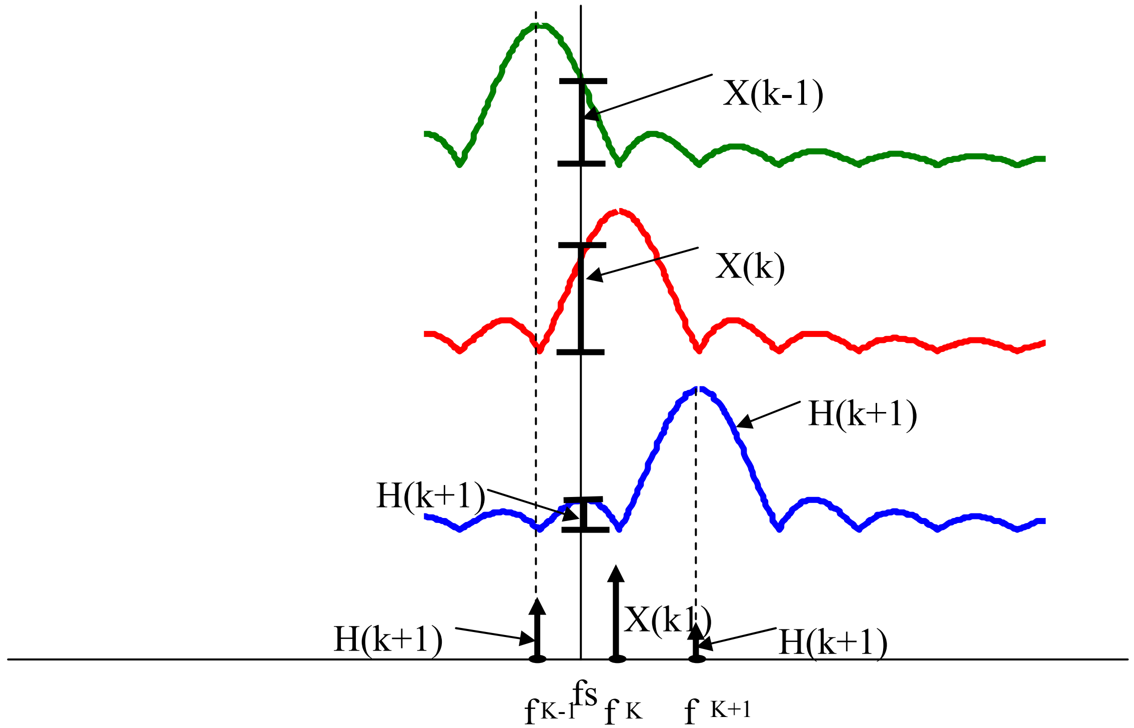




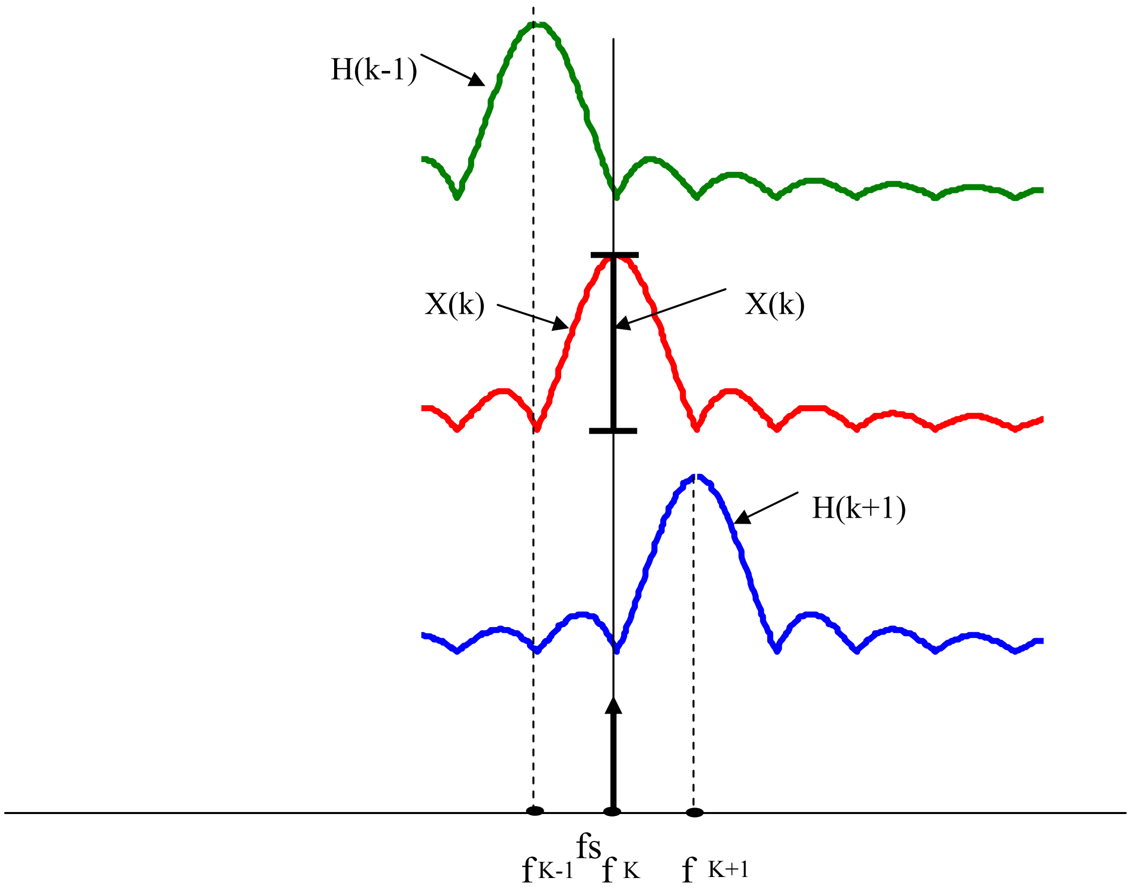
Signal Processing: Spectral Analysis

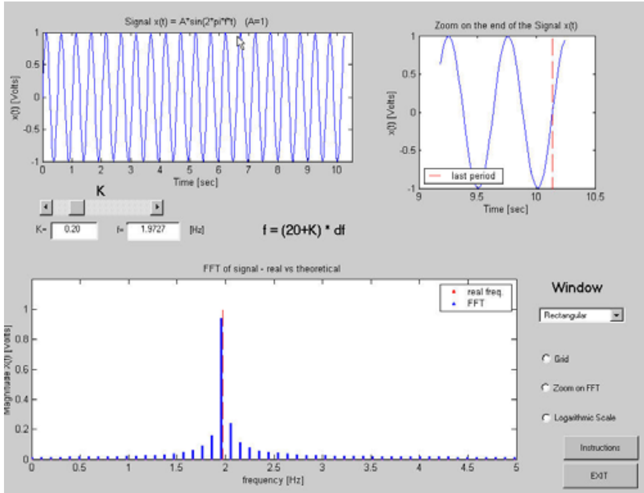


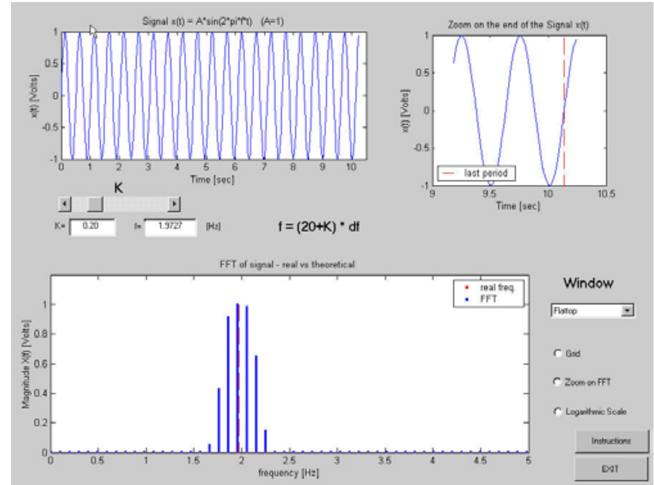
Signal Processing: Spectral Analysis

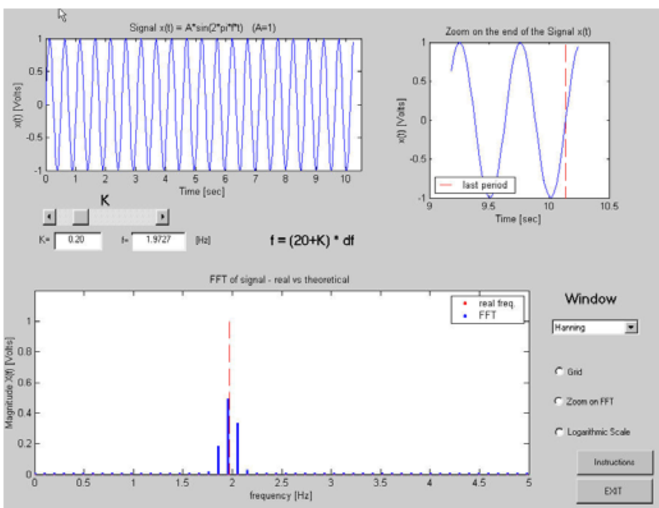


Signal Processing: Spectral Analysis


H(k-1)
X(k)
X(k)
H(k+1)
fs
f K-1
f K
f K+1

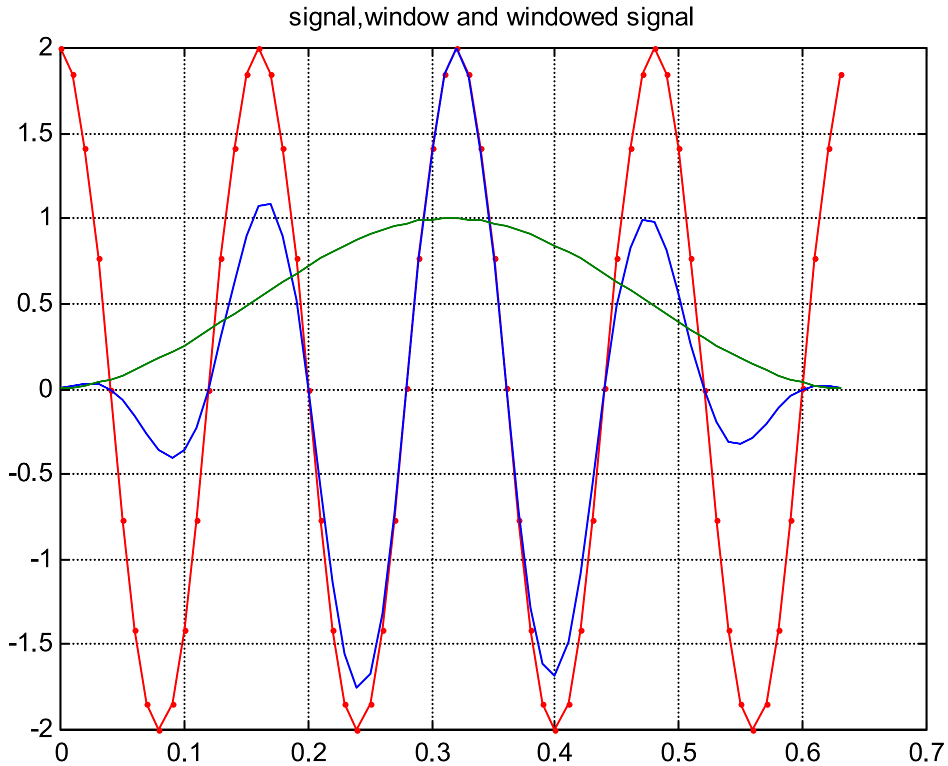






Signal Processing: Spectral Analysis

Give diminishing weights to the signals beginning and end by means of suitable windows.



The windowing is undertaken by multiplying the time signal by the appropriate window function.

$$x'(i)=w(i)x(i)$$