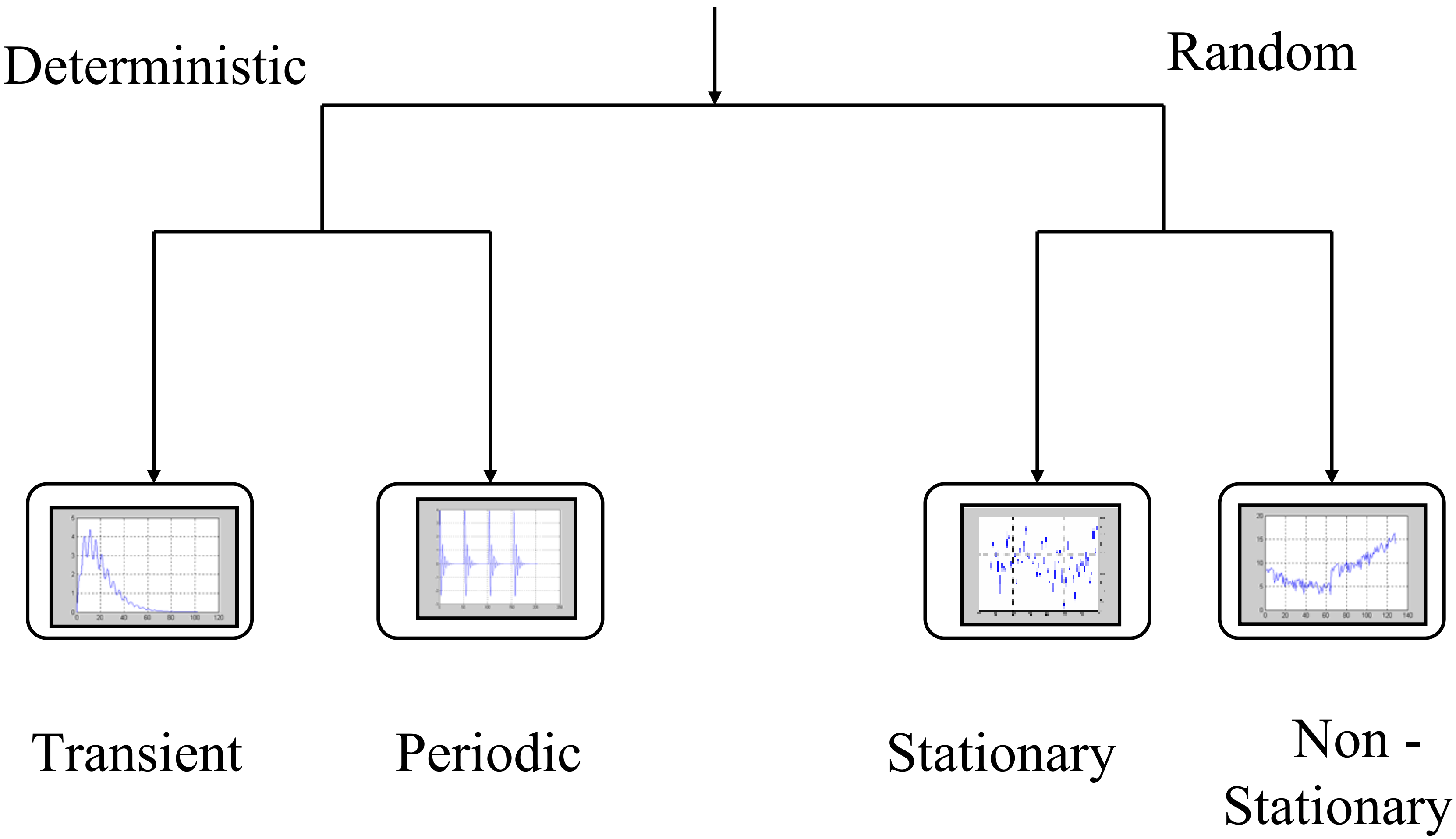
SIGNALS

The characterization as well as analysis methods depends on the signal structure.
The following are some classification possibilities.

Deterministic vs. random
Transient vs. continuous
Stationary vs. nonstationary

In practice we often encounter combinations of signal types.
An example would be a harmonic signal contaminated by random noise.

Classification of Signals



Descriptions

Transient signals - *energy*

This is defined as

$$E = \int_{0}^{T} x^2(t)\,dt$$

where T is the signal duration.

The energy is finite for a signal limited within an interval T.

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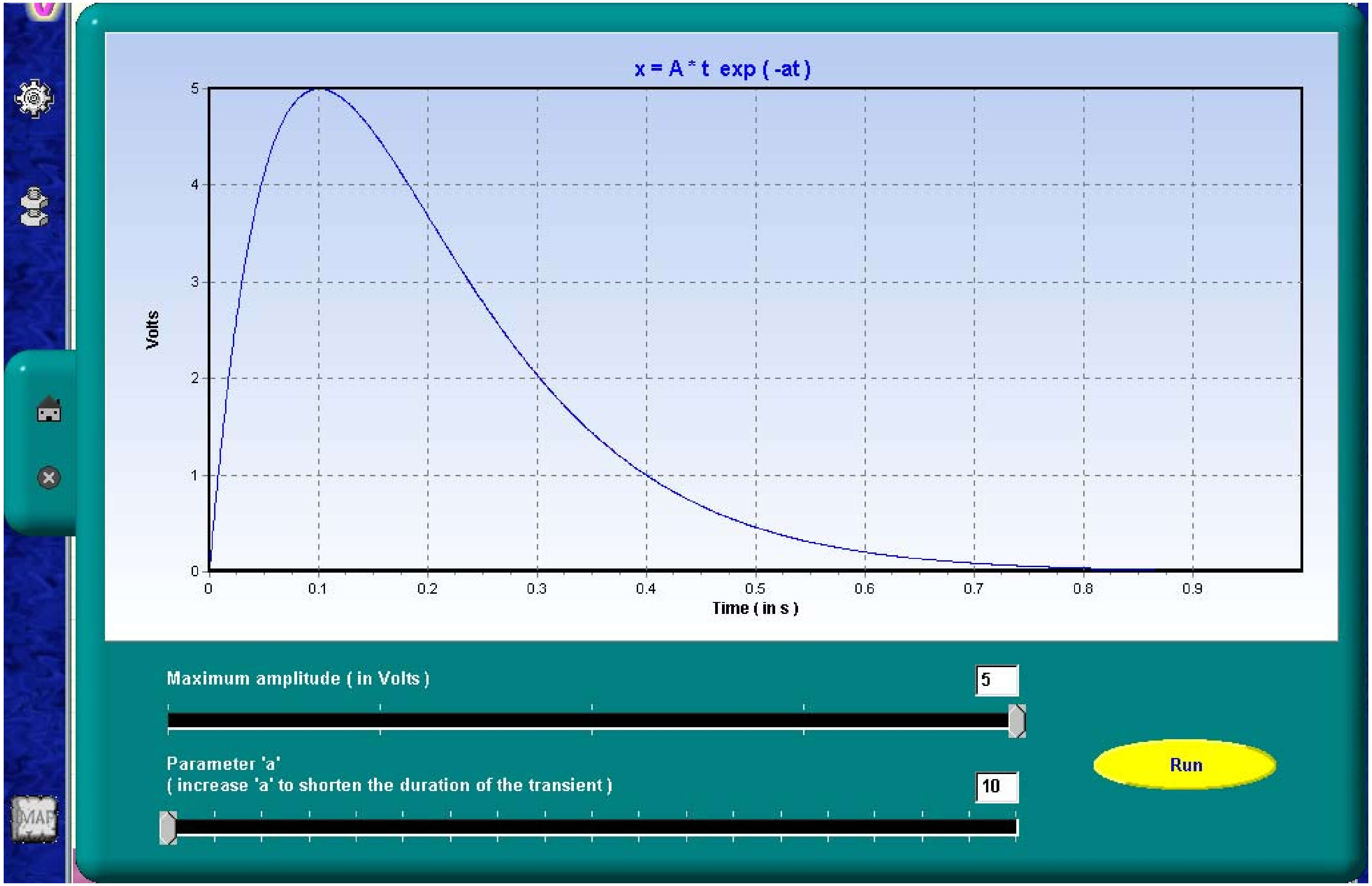
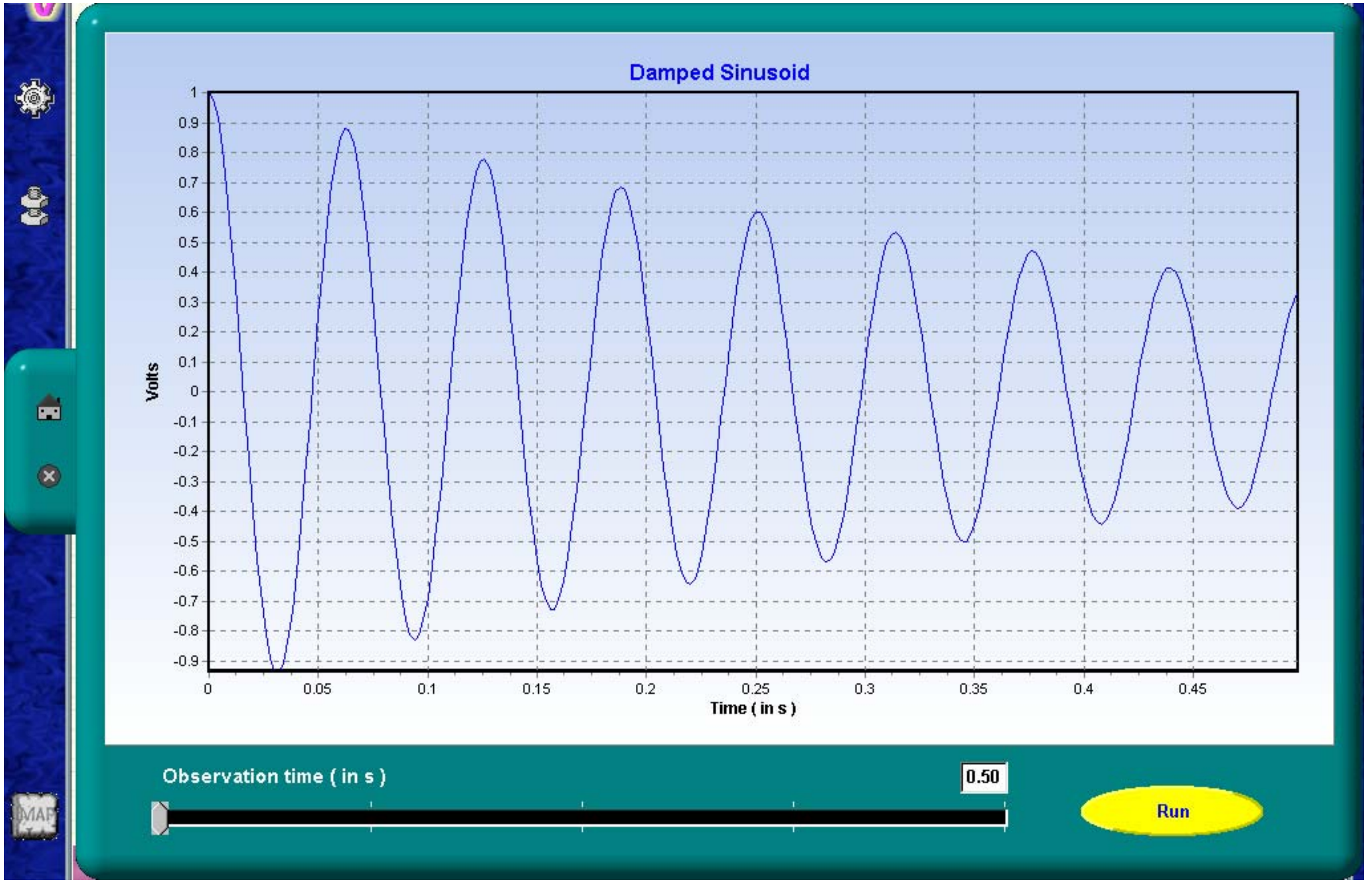
The units are

E [V^2 - sec, G^2 - sec...]

and such a signal is also called an energy signal.



Signal Processing: Signals



Signal Processing: Signals

Continuous signals - *power*

For such a signal

$$E \to \infty$$

as

$$T \to \infty$$

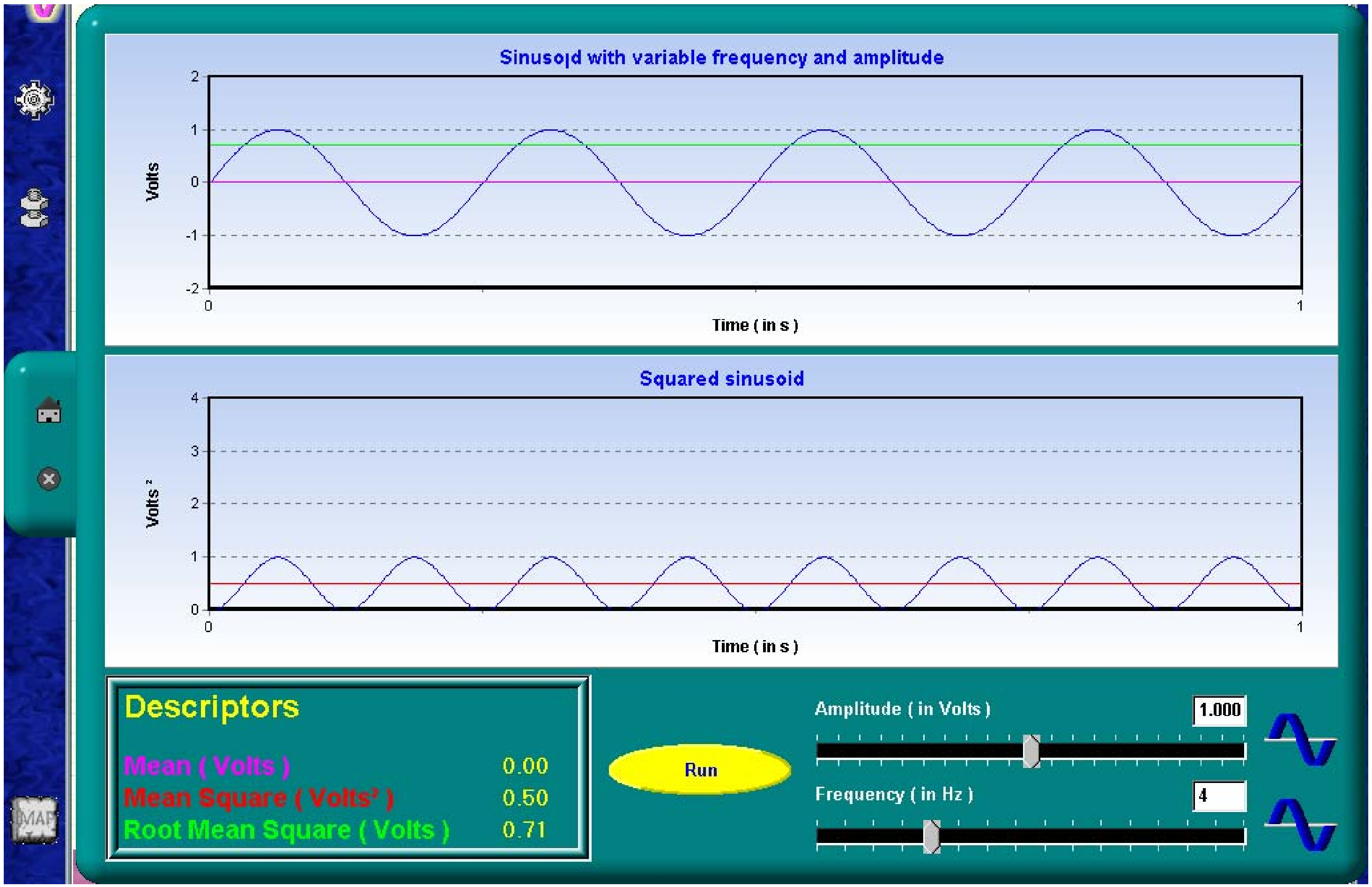
and power can be used instead of energy.

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

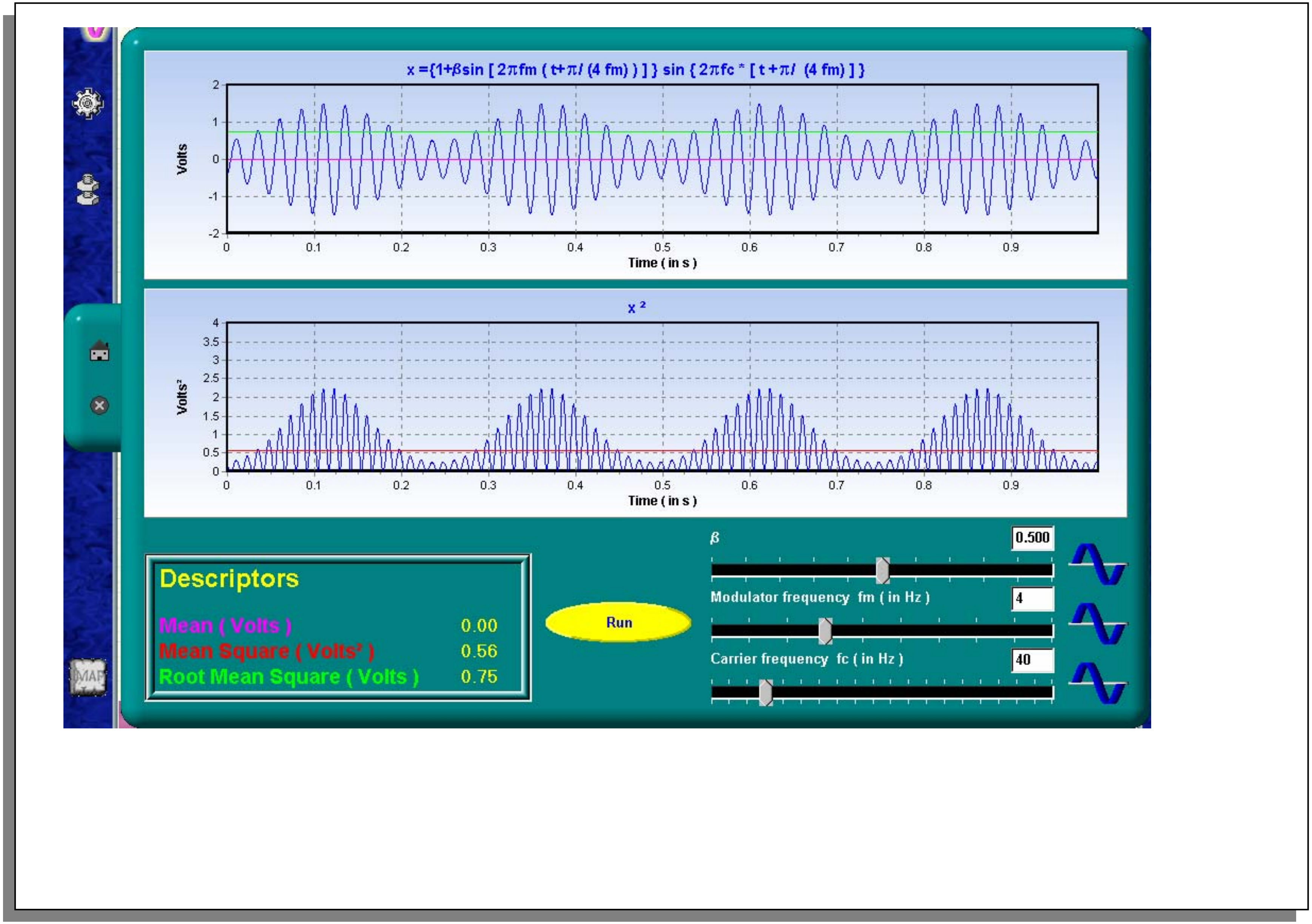
and P exists for $T \to \infty$ (but not E). The units are

P [V^2 , G^2 ...]

and such signals are called power signals.

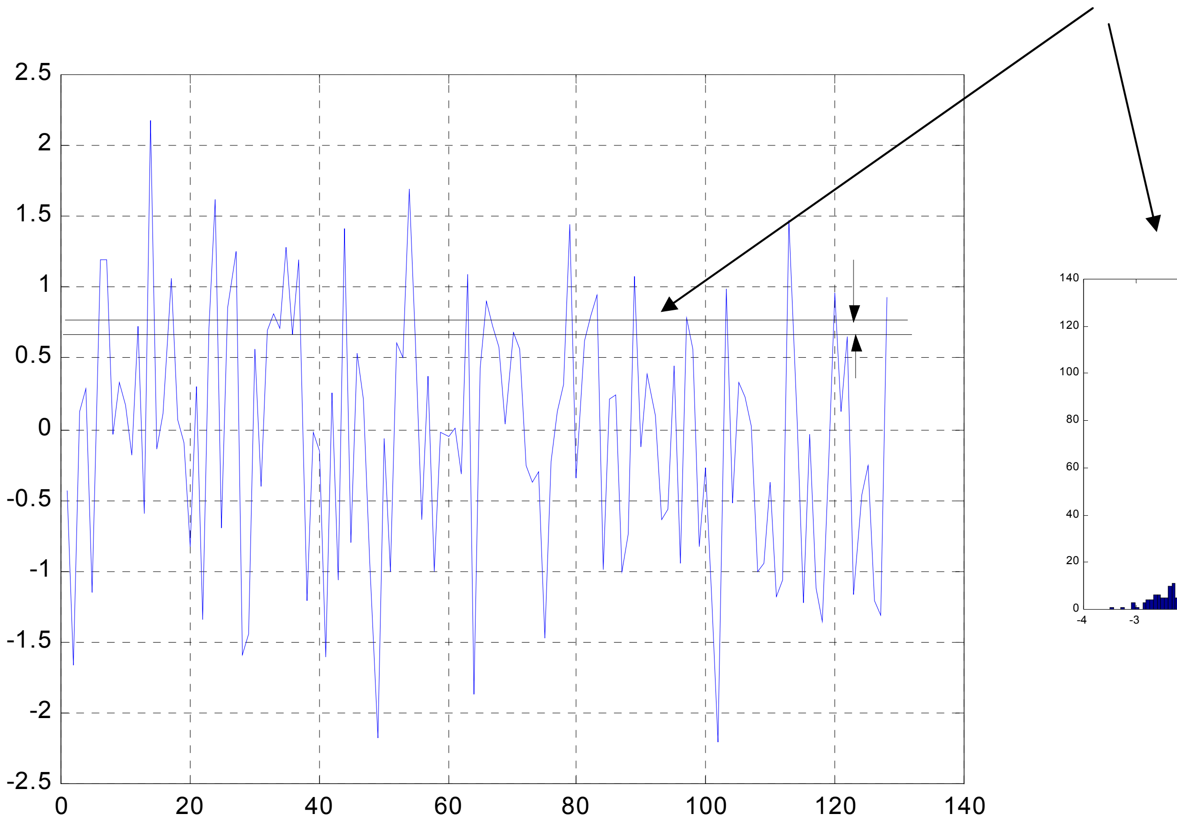


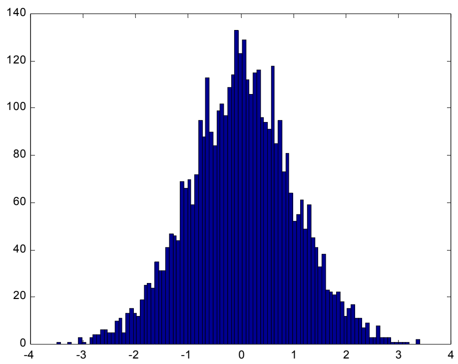
Signal Processing: Signals



Signal Processing: Signals

Percentage of time within window





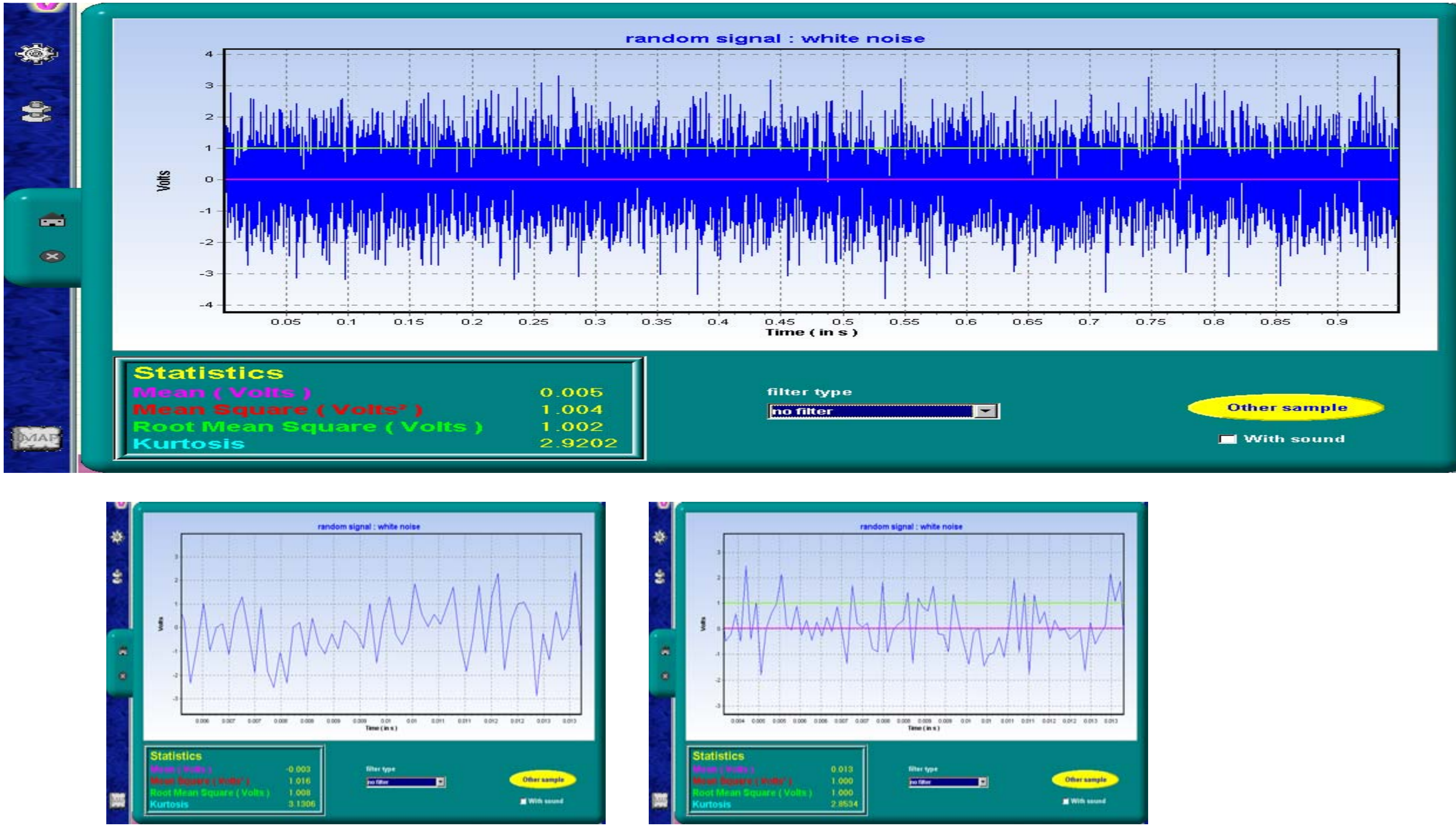
Random signals

While specific signal shapes can define deterministic sig
only statistical properties can describe random signals.
Probabilities can be defined as percentage of time for which a
n a specific amplitude range. A histogram (discrete) or probab
density function p(x) (continuous) can be defined. Thus

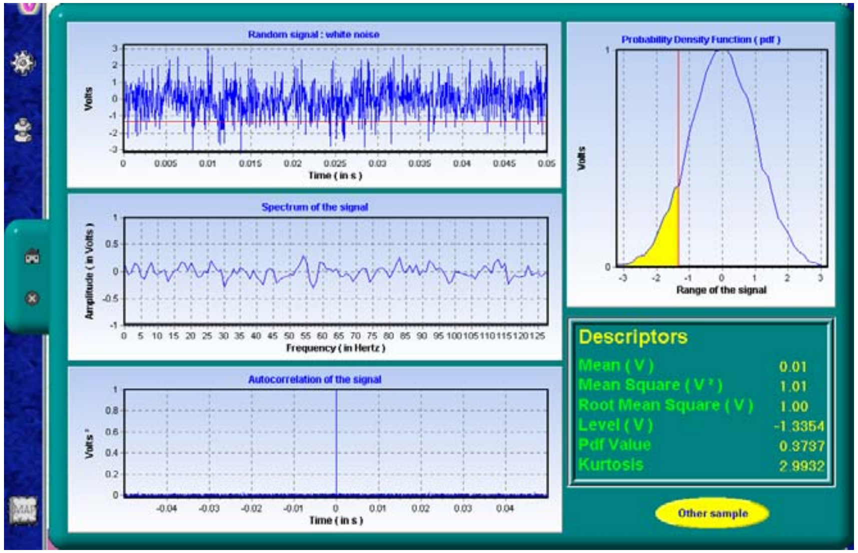
$$p(x) = \lim_{\Delta x \to 0} \frac{\mathrm{Prob}\left[x < x(t) < x + \Delta\lambda\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\lim_{T \to \infty} \frac{\sum T_i}{T}\right]$$

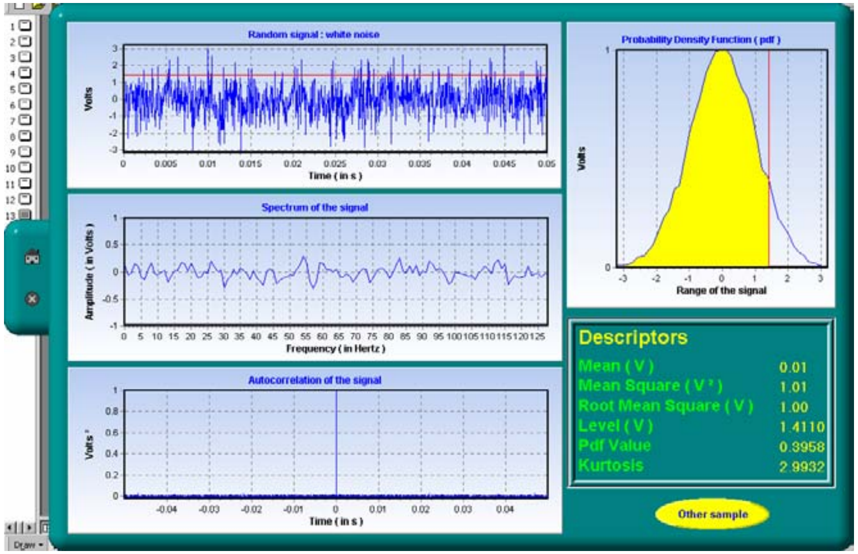
here Ti are the intervals where the signal lies in the amplit
window between x and x + Δx.

The area under p(x) is set to 1 for normalization.
Then the area under p(x) is the percentage of time
the signal is in the corresponding range of x.



Signal Processing: Signals





Signal Processing: Signals

Signal parameters can be based on p(x). Statistical moments μ_k are

$$\mu_k = E\left[x^k\right] = \int_{-\infty}^{\infty} x^k p(x) dx$$

where E[·] denotes expectations.

The first and second moments are called "mean" and "mean square"

M(mean), $\mu_1 = \int_{-\infty}^{\infty} x p(x) dx$

MS(mean square):

$$E\left[(x - \mu_2)^2\right] = \int_{-\infty}^{\infty} (x - \mu_1)^2 p(x) dx \rightarrow \int_{-\infty}^{\infty} x^2 p(x) dx$$

Often central moments, around the mean, are used. This is especially convenient for vibration signals, where the mean is set for zero by the measurement process.

The second central moment is called variance

Variance:

$$E\left[(x-\mu_2)^2\right] = \int_{-\infty}^{\infty} (x-\mu_1)^2 p(x)dx \rightarrow \int_{-\infty}^{\infty} x^2 p(x)dx$$

and is square root is the standard deviation σ, hence

σ^2 = variance

For signals, moments are computed via time averages

$$\mu_1 \equiv \mu = \frac{1}{T}\int_0^T x(t)dt \rightarrow 0 \text{ for vibrations}$$

$$T \rightarrow \infty$$

$$\sigma^2 = \frac{1}{T}\int_0^T x^2(t)dt$$

$$T \rightarrow \infty$$

Hence σ^2 (the variance) is also called the Mean Square - MS, and σ the Root Mean Square - RMS.

For a random signal with zero mean,

$$\text{RMS} \equiv \sigma$$

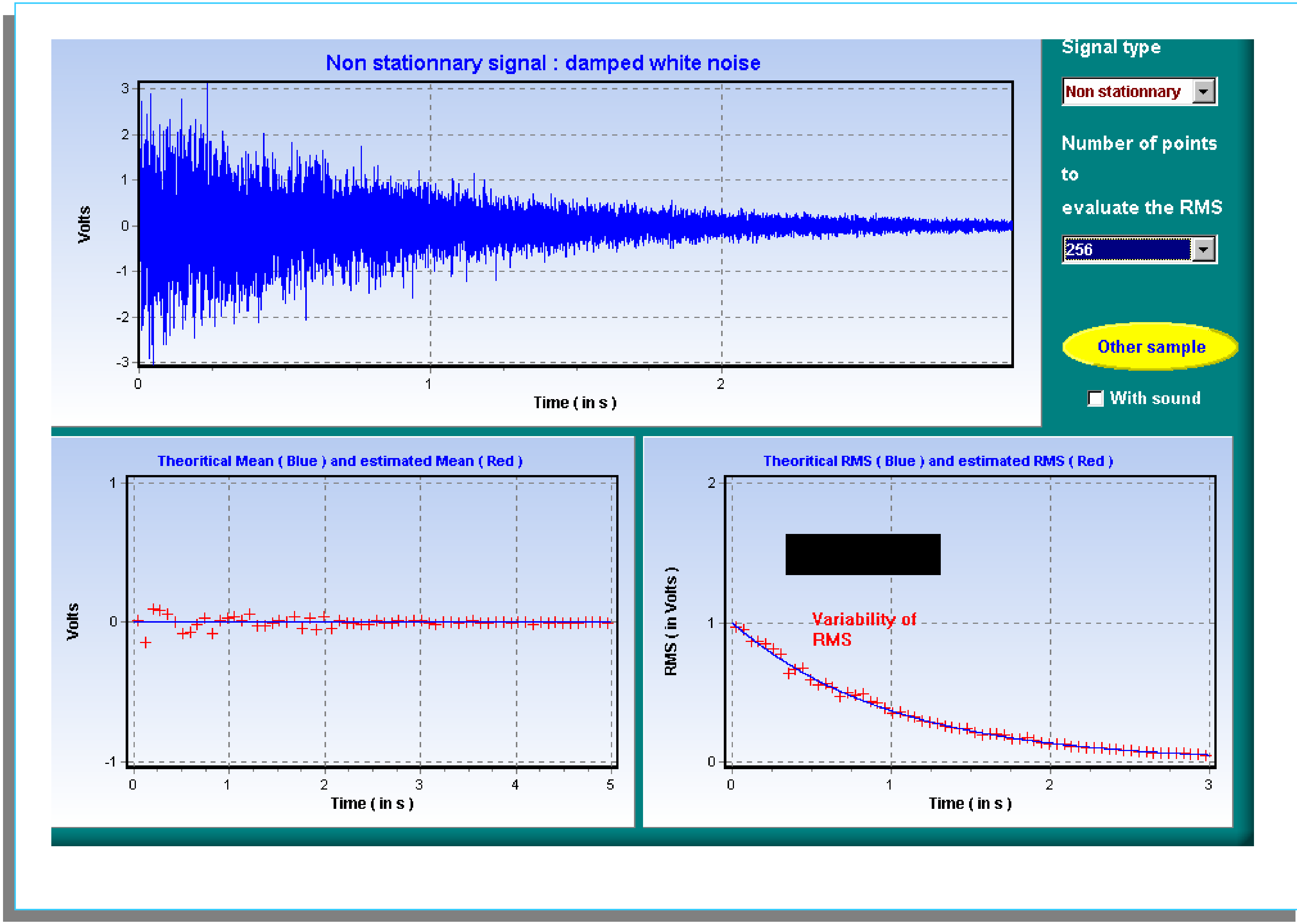
Many random phenomena have distributions which approximate the Gaussian distribution, also called the Normal distribution:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2} \right] \equiv N[\mu, \sigma]$$

p(x) is described by two parameters only, the mean μ and the variance σ^2. The spread (width) of this bell shaped function depends on σ. A normalized function is defined by

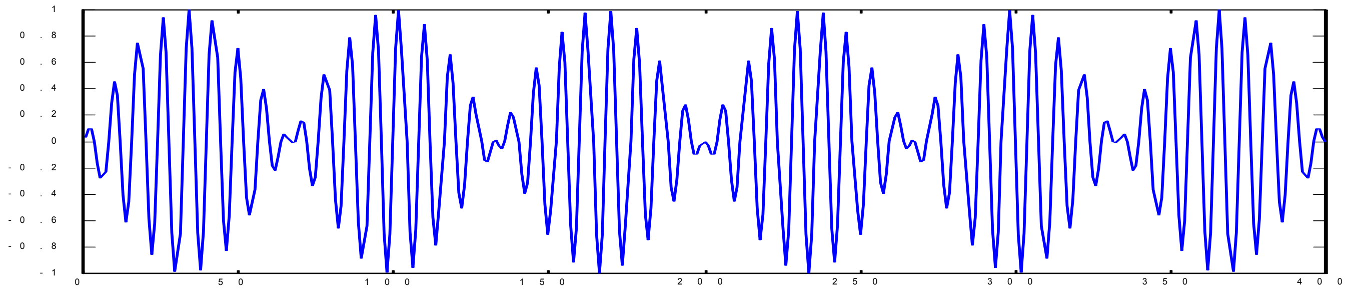
$$N[0,1] \equiv p(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-x^2/2\right]$$

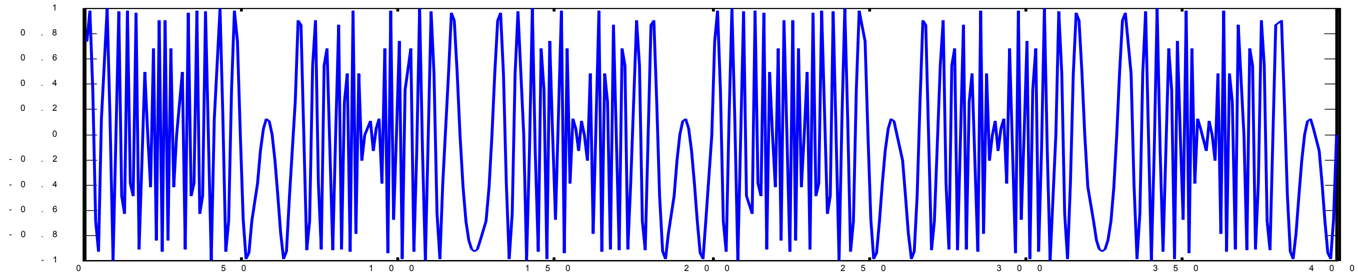
Such a signal is practically enveloped within $\pm 3\sigma$.



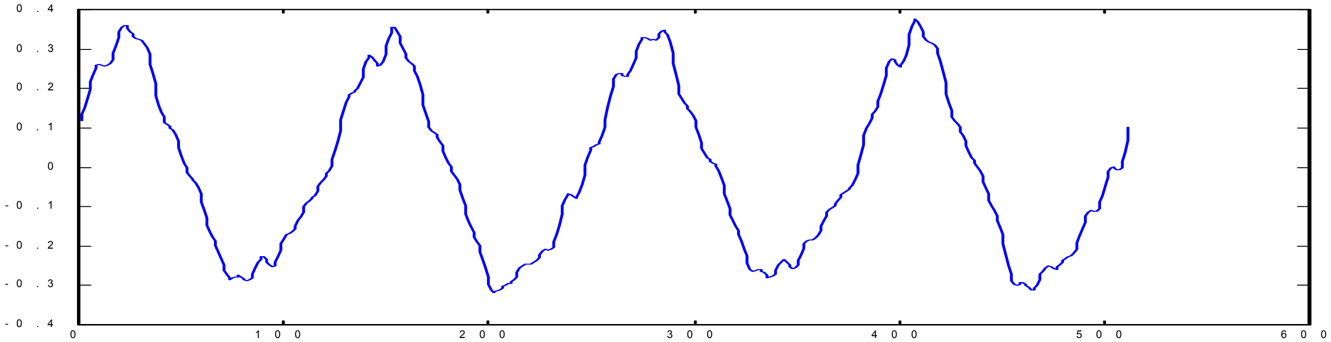
Signal Processing: Signals

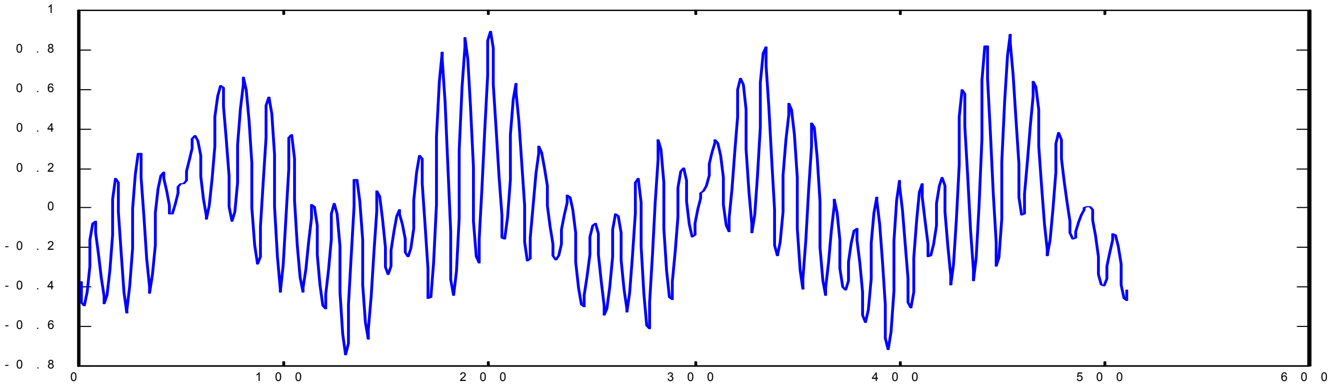
Modulations: Periodic



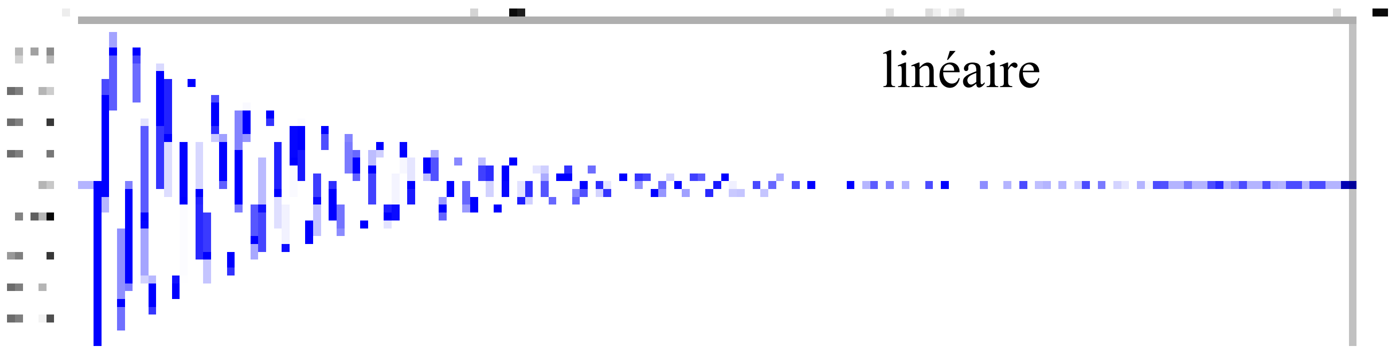


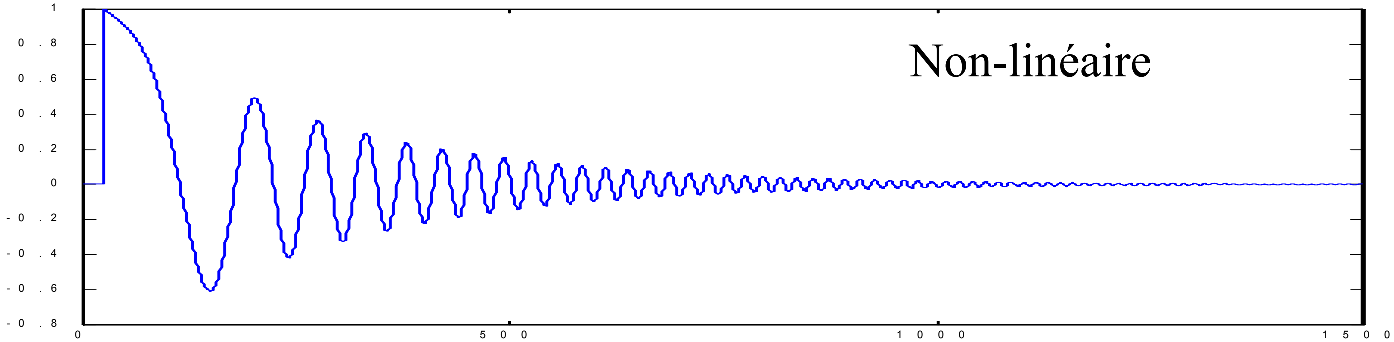
Vibrations of rotating machines



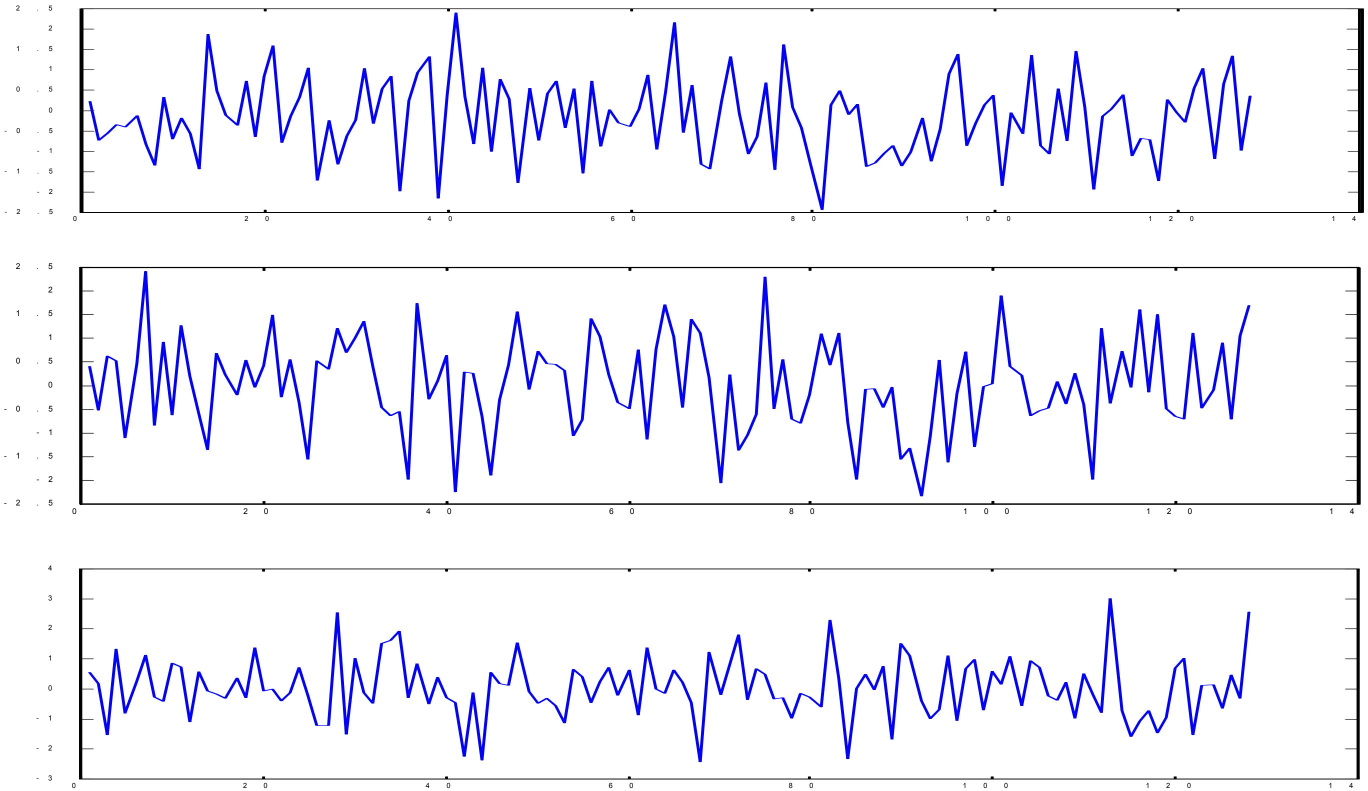


Impulse response of SDOF system

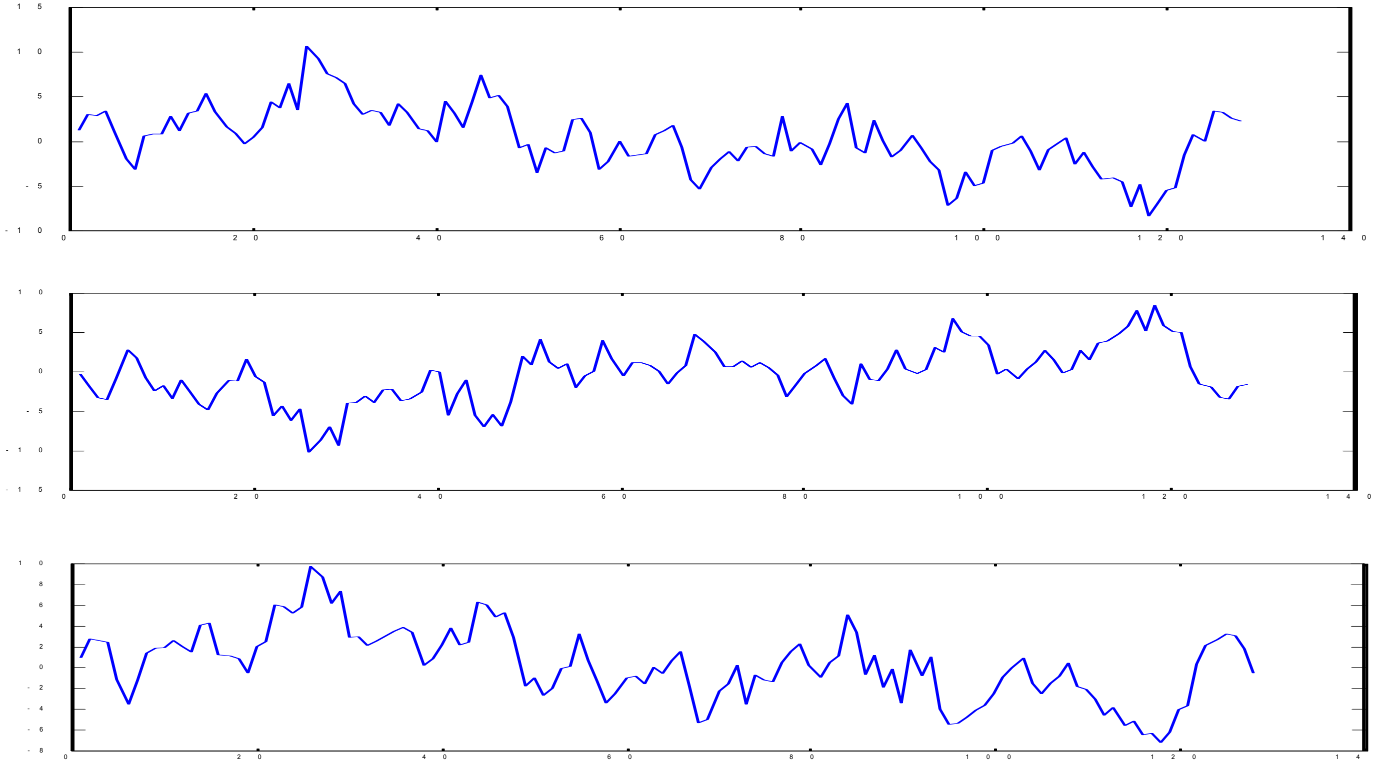




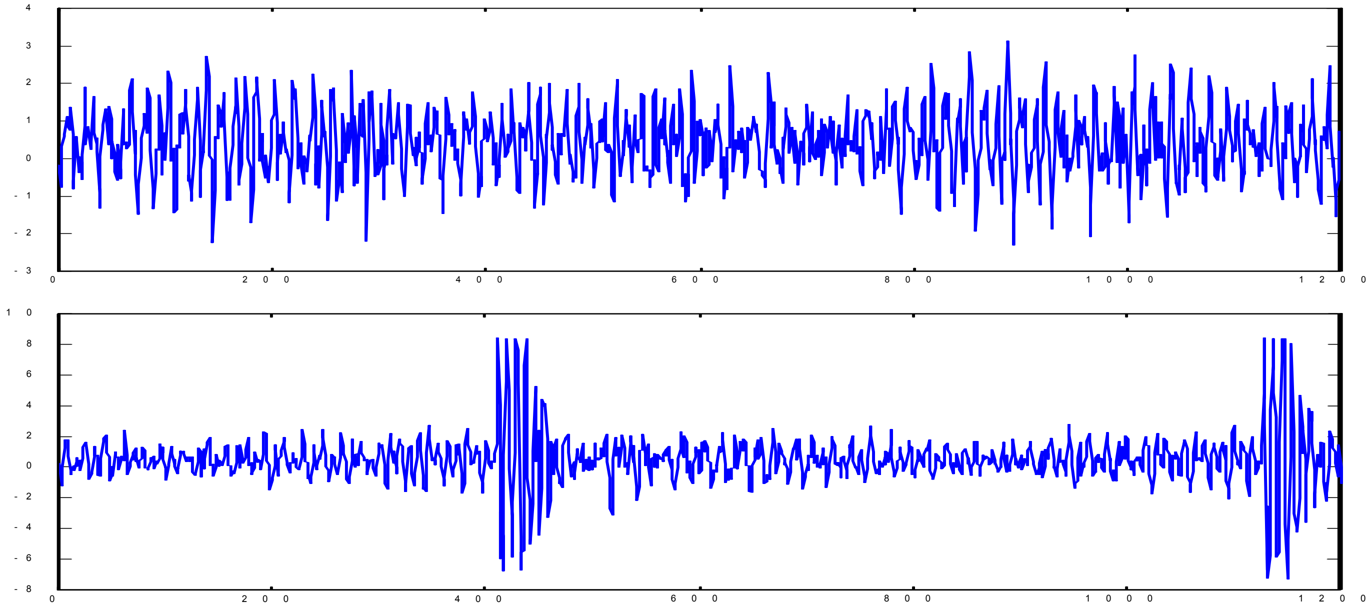
White noise: 3 realizations



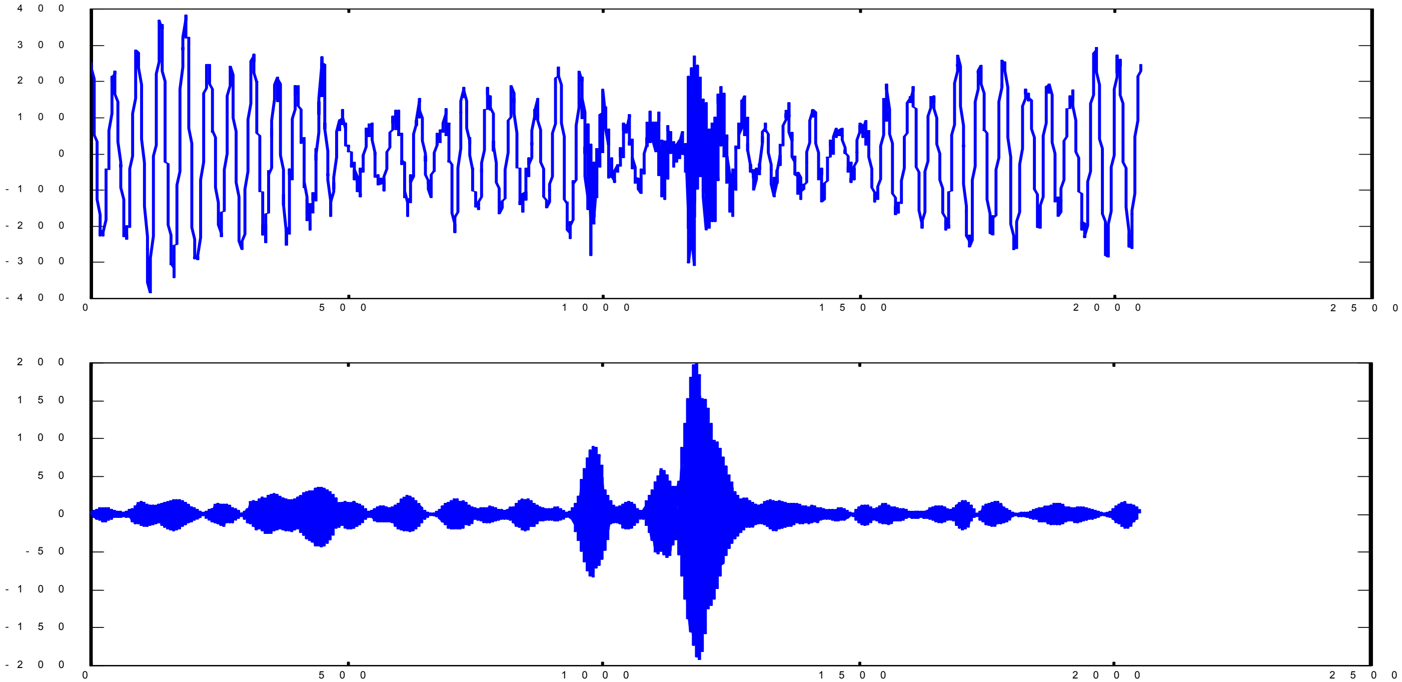
- Response of system excited by white noise



- Gear system:
- Good and faulty gear

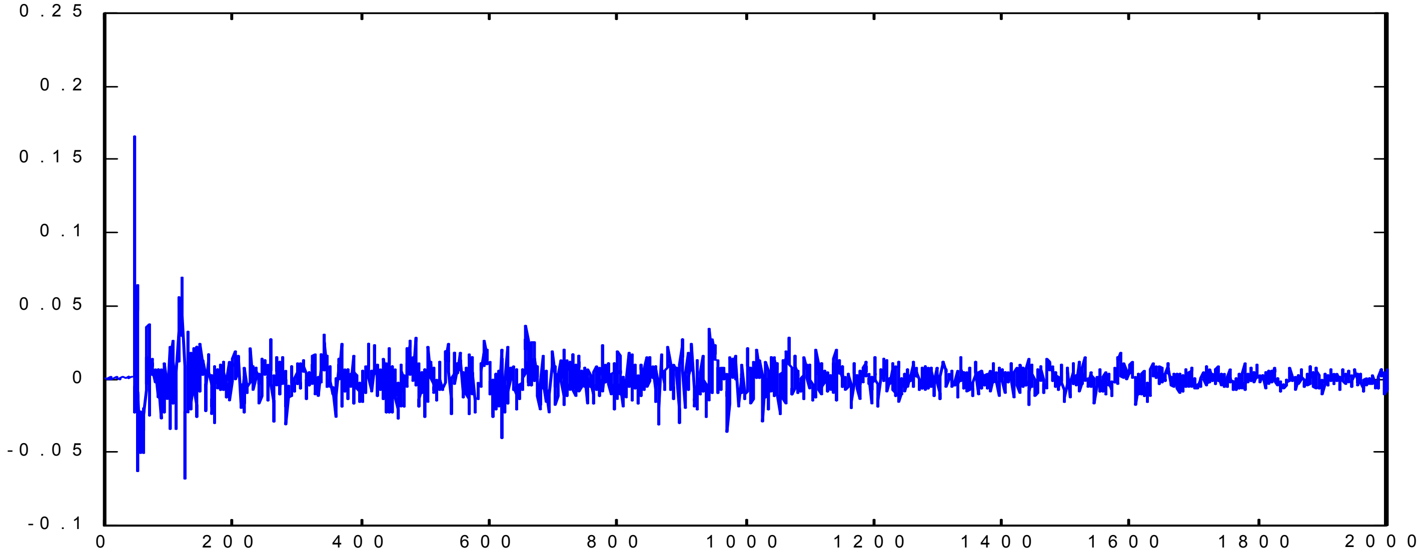


Monitoring of tool wear



Signal Processing: Signals

- Acoustic impulse response of room



Biomedical signals

Evoked potentials , response to flash (measured on scalp)