Instabilities and subsequent non-equilibrium pattern formation is often encountered in spatially extended systems. If one of the space dimensions is distinguished for physical or geometrical reasons, dimension-reduced models may be derived successfully by integrating out that dimension. Here we consider (thin) liquid films on a horizontal substrate where the particular dimension is the vertical one.

In fluid mechanics, the nowadays called 'Shallow Water Equations' were historically the first dimension-reduced set derived by Adhemar Jean Claude Barre de Saint-Venant already in 1871, allowing for the theoretical and later on numerical linear and nonlinear computations of long surface gravity waves on rivers, lakes or oceans if viscosity can be neglected.

Another way of dimension reduction is obtained by enslaving many temporally fast modes by only a few slow modes normally present in the neighborhood of instabilities. The celebrated Swift-Hohenberg equation (Jack Swift and Pierre Hohenberg, 1977) and its extensions derived in the 80ies and 90ies of the 20th century describes qualitatively well the behavior of several pattern forming convection instabilities also including effects of a free surface.

In thin liquid films, normally viscosity is the dominating mechanism and inertia can be neglected (zero-Reynolds-number limit). Fluid velocity is enslaved by the film profile and a single 2D partial differential equation for the film thickness is derived, as first demonstrated by Agienus Vrij in 1966.

However, there are cases even in thin films where the Reynolds number can be large (falling films, mechanically vibrated substrates) and inertia must be kept. This leads to extended thin-film equations where the dynamics of the fluid flow occurs as additional degree of freedom.

In this talk, the situation of parametrically excited films by mechanical vibrations will be studied in more detail. Linear stability as well as nonlinear results of the reduced model will be presented and compared to findings of the full set of hydrodynamic basic equations. Nonlinear results of the reduced model show interesting new patterns like confined pulses or quasi-periodic geometries. Finally, a recently derived model describing a two-layer thin film system is presented.