Nonlinear dynamics of discrete mechanical systems with flat dispersion bands

The seminar will be given in Hebrew

Physical lattices with configurational local symmetries possess interesting features, an apparent one being the existence of flat dispersion curves (bands) in the linear spectrum. Such flat bands may be associated with the existence of hidden dynamic modes, which cannot be excited externally in the linear regime. An additional phenomenon associated with the flat bands is the emergence of detached, spatially localized, perfectly compact dynamic modes, which can have non-trivial stability characteristics in the nonlinear case. Many examples of lattices with flat bands were discussed in the recent literature, among others photonic lattices with Kerr nonlinearity, chains of coupled pendula, granular systems with Hertzian contact, Bose-Einstein condensates, and DNA molecules. One interesting example of a nonlinear lattice with a flat band is a discrete mechanical system of masses and springs, where in addition to linear links between the masses, there are displacement limiters producing impacts. Piecewise linear systems of this sort have a noteworthy advantage. The stability of periodic solutions in piecewise linear systems can be analyzed with high accuracy and efficiency through analytic construction of the monodromy matrix, which simplifies the study of local bifurcations. On the other hand, the impacts render such systems strongly nonlinear, which makes the study of global bifurcations by averaging techniques, such as canonic transformations, much harder. Recently introduced theoretical approaches, such as the application of the Action-Angle formalism in conjunction with the notion of the Limiting Phase trajectory enable the execution of analysis of global bifurcations of small non-integrable piecewise linear systems. In this seminar, the results of a theoretical PhD study on the subject of nonlinear dynamics of discrete mechanical systems having flat dispersion bands will be presented in accordance with the aforementioned ideas: hidden modes in smoothly nonlinear systems, including the development of a suitable local stability analysis method; compact modes, including analytic local stability results for a piecewise linear non-smooth system; and global analysis and prediction of critical transitions in a strongly nonlinear (non-smooth) case.