The overall objective of our work is developing understanding of the complex flow-turbulence interactions occurring in the propagation of premixed flames. The determination of the turbulent flame speed in this context is of great practical importance providing, for example, the mean fuel consumption rate in a combustor operating under turbulent conditions. Early phenomenological and empirical studies resorted to geometrical and scaling arguments to deduce expressions for the turbulent flame speed. The common denominator of these expressions is an increase in speed associated with the increase in flame surface area. The current work is based on an asymptotic model that exploits the disparity between the distinct scales associated with the flow field, the diffusion processes, and the highly temperature-sensitive reaction rates. In this model, the flame is confined to a surface that separates burned from unburned gases and propagates relative to the fresh combustible mixture at a speed that depends on the local mixture and flow conditions. The turbulent hydrodynamic field is modified, in turn, by the gas expansion resulting from the heat released during combustion. A parametric study has been carried out for mixtures with positive Markstein length (i.e., absent of thermo-diffusive instabilities), examining different factors including flame stretching, the Darrieus-Landau instability, flame folding and pocket formation, all of which are ubiquitous in turbulent propagation, with nontrivial implications. Scaling laws for the turbulent flame speed are proposed for low-to-moderate turbulence intensities that highlight the dependence on physically measurable quantities. The results, devoid of turbulence-modeling assumptions and/or ad-hoc coefficients, can help explaining the influence of varying the system parameters individually and collectively, and formulating physically based small-scale models for large-scale numerical simulations of turbulent flames.