In the past few decades, locally resonant discrete acoustic structures have drawn considerable attention for their unique and adaptive material properties. Liu et al. first introduced the concept of locally resonant meta-materials (LRSM). The LRSM is composed of inclusions of dense material shapes covered with a soft material, with the inclusions embedded in the matrix. Some intriguing dynamical properties of this special class of meta-materials was revealed in the recent studies. In most of these works, the locally resonant sonic structures assumed a perfectly linear internal resonator. However, inclusion of nonlinear components may drastically change the dynamical properties of the locally resonant structure, leading to the well-known phenomena of wave localization, direction-reversing waves, etc. The present study is comprised of two main parts. In the first part, we analyze the 1D purely nonlinear (cubic nonlinearity), locally resonant chain of elements. Each element of the chain incorporates a purely nonlinear local attachment. This state of dynamical model is usually referred to as a sonic vacuum. This part of the study is devoted to the analysis of several families of special localized solutions and their corresponding bifurcation structure. In the second part of the study, we consider the dynamics of locally resonant unit-cell model mounted on a 2D nonlinear local potential, incorporating an internal small-mass oscillatory structure. In this part of the study, we have focused on the analysis of special regimes of energy channeling, manifested by the intense and mild, resonant energy transport from horizontal to vertical vibrations of the outer unit. Special emphasis in this part of the study is given to the analysis of the mechanism of formation and destruction of these regimes emerging in the 2D structure, subject to impulsive and self-induced excitations. Results of numerical simulations are in agreement with these of the analytical models.