Solitary waves in nonintegrable lattices

The seminar will be given in English.

The interplay between discreteness and nonlinearity in many physical systems leads to the formation of solitary waves. For example, such waves were experimentally observed in granular materials, electrical transmission lines and optical fibers. Much of the interest in these nonlinear waves was triggered by the pioneering study by Fermi, Pasta and Ulam (1955). The subsequent work of Zabusky and Kruskal (1965) has revolutionized the nonlinear science by connecting the FPU problem to its quasicontinuum near-sonic limit described by the KdV equation. In integrable systems solitary waves, known as solitons, are now well understood, with one-dimensional Toda lattice being the most prominent example that has an exact solution covering a broad range of behaviors from delocalized low-energy waves in the KdV limit to highly localized high-energy waves. Most discrete systems, however, are nonintegrable. In this case understanding the transition from the KdV limit to the strongly discrete waves has mostly relied on numerical and quasicontinuum approximations.

In this talk I will review some of these results and describe recent work with Lev Truskinovsky on a nonintegrable FPU problem with a piecewise quadratic potential. We construct an exact solitary wave solution that captures the entire crossover velocity range between the low-energy limit and strongly localized waves that involve only one particle moving at a time. The solution is expressed in the form of an infinite series. A truncation of the series involving progressively smaller characteristic wavelengths produces a nested set of approximate solutions. Even the simplest solution of this type that accounts only for the longest wave lengths provides a better overall approximation of solitary waves than some conventional quasicontinuum models. Recent extensions of this work to a two-dimensional lattice and to the problems involving competitive second-neighbor interactions and nonconvex potentials will also be discussed, along with some related open questions.