Interaction between microfluidic droplets in a Hele-Shaw cell:
closely spaced droplets and a droplet within a droplet

Various fluidic systems, such as chemical and biological lab-on-a-chip devices, involve motion of multiple droplets within an immersing fluid in a narrow micro-channel. Modeling the dynamics of such systems requires the calculation of the forces of interaction between the moving droplets. These forces are commonly approximated by superposition of dipole solutions, which requires the assumption of sufficiently large distance between the droplets. In this work we obtain exact solutions for two droplets, and a droplet within a droplet, located within a moving immersing fluid and without limitation on the distance between the droplets. This is achieved by finding the solution of the Laplace equation for the pressure in a bi-polar coordinate system and transformation and calculation of the force in a Cartesian coordinate system. Our results are validated with numerical computations, experimental data and with the existing dipole-based models. We utilize the results to calculate the dynamics of a droplet within a droplet, and of two close droplets, located within an immersing fluid with an oscillating flow velocity. The obtained results may be used to study the dynamics of dense droplet lattices, common to many current microfluidic systems.