Nonlinear Dynamics, Bifurcations and Chaos in Magnetic Resonance Force Microscopy

Magnetic resonance force microscopy (MRFM) is an imaging technique that enables acquisition of three-dimensional magnetic images at nanometer scales, and has been adapted for detection of magnetic spin of a single electron. It is based on combining the technologies of magnetic resonance imaging (MRI) with atomic force microscopy (AFM). In conventional MRI devices the electronic spins are detected by measuring their magnetic induction using an inductive coil as an antenna. However, in MRFM the detection is implemented mechanically using a cantilever to directly detect a modulated spin gradient force between the sample spins and a ferromagnetic particle attached to the tip of the cantilever. While MRFM systems are receiving a growing amount of interest, to date, a comprehensive theoretical treatment is still lacking. Existing models are based on simplistic lumped-mass reductions that include linear estimates of cantilever stiffness and damping complemented by a nonlinear approximation of the magnetic force and are unable to resolve the spatio-temporal complexity of the magneto-elastic sensor.

We thus consistently formulate a nonlinear initial-boundary-value problem (IBVP) combining the three-dimensional motion of the micro-cantilever and the dynamic interactions of the spin magnetic moments. We reduce the IBVP to a nonlinear dynamical system and investigate stability of the equilibrium solution for different values of a magnetization parameter to reveal both transcritical and saddle-node bifurcations. A numerical and approximated analytical Melnikov-Holmes stability bounds are also found. We use an asymptotic multiple-scale analysis to estimate analytically the cantilever frequency shift and to derive slowly-varying evolution equations for several cases of internal resonances. Numerical integration of the dynamical system for different values of parameters reveals quasiperiodic and non-stationary motion which can enable simultaneous multi-functional sensing of topography and material properties of magnetized samples.