Whispering-gallery-mode resonances in optical spheroidal cavities interacting with a subwavelength particle

Optical whispering-gallery-modes (WGM) are electromagnetic excitations of axially symmetrical dielectric bodies arising due to total internal reflection. When excited by an external light they manifest themselves as resonances with exceptionally high quality factors, which makes them very sensitive to small changes in their environment. Therefore, WGM resonators have attracted a great deal of attention for sensing and sizing of individual small particles adsorbed onto their surface. Despite significant experimental progress resulted in a single protein detection by a nominally spherical resonator, an adequate theoretical understanding and efficient methods for numerical modeling of the resonator-nanoparticle system are still lacking. This is not surprising because the particle destroys the axial symmetry of the system and the resulting three-dimensional problem becomes intractable by direct numerical methods. In this paper we develop an ab initio theory of the WGM resonances of a spheroidal resonator interacting with a subwavelength particle treating the particle in the dipole approximation. Using this approach we construct an efficient computational method allowing for accurate numerical simulations of the resonances in such a system. Our approach also provides approximate analytical expressions for frequencies and widths of the particle-modified resonances, which agree well with numerical results and afford a deeper insight into the role of symmetry and the degeneracy of the initial WGM resonances in the resonator-particle interaction.