Experimental Modal Analysis

Conduct test excitation/response

Compute transfer functions FRFs

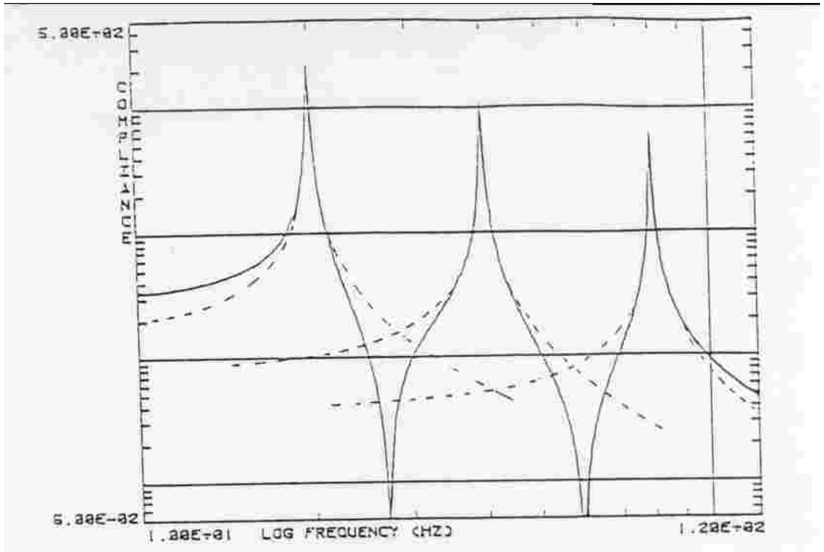
Extract modal parameters from FRFs

$$H_{ik} = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{kr}}{\omega_r^2 - \omega^2 + j2\varepsilon_r\omega_r\omega} = \sum_{r} \frac{{}_{v}A_{ik}}{\omega_r^2 - \omega^2 + j^2\varepsilon_r\omega_r\omega}$$

$${}_{r}A_{ik} = \phi_{ir}\phi_{kr}$$

$$H_{ik} = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{kr}}{\omega_r^2 - \omega^2 + j2\varepsilon_r\omega_r\omega} = \sum_{r} \frac{{}_{v}A_{ik}}{\omega_r^2 - \omega^2 + j^2\varepsilon_r\omega_r\omega}$$

$${}_{r}A_{ik} = \phi_{ir}\phi_{kr}$$



SDOF - Peak method

$$H_{ik} = \frac{{}_r A_{ik}}{\omega_r^2[1-(\omega/\omega_r)^2 + 2j\xi_r\omega/\omega_r]}$$

ω_r where H peaks

ξ_r 3 db frequencies ω_2, ω_1

$$\xi_r = \frac{\omega_2 - \omega_1}{2\omega_r}$$

Assuming $\xi_r << 1, \quad \omega \approx \omega_r$

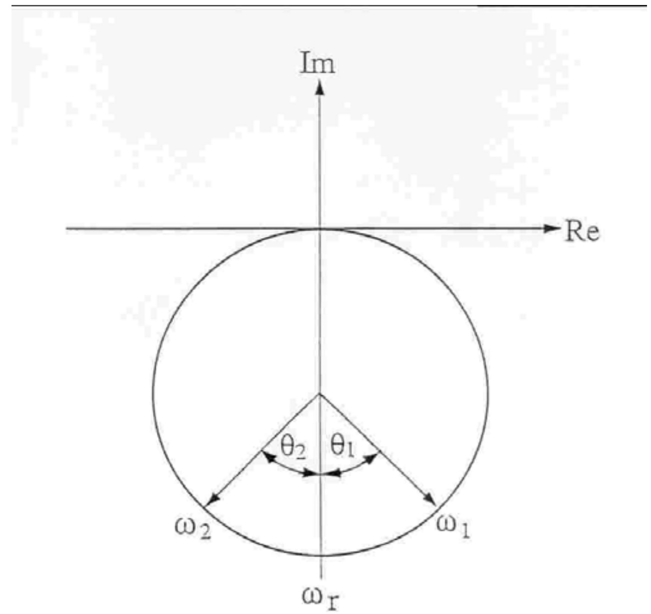
$$|H(\omega_r)| = \frac{{}_r A_{ik}}{2\xi_r\omega_r^2}$$

SDOF - circle method

ω_r maximum angular spacing

$$\xi_r = \frac{\omega_2 - \omega_1}{\omega_r\left[\tan\left(\frac{\theta_1}{2}\right) + \tan\left(\frac{\theta_2}{2}\right)\right]}$$

$$Dr = \frac{{}_rA_{ik}}{2\xi_r\omega_r^2}$$



Note: Frequencies, damping – global

Shapes - local

MDOF - complex exponentail fit

$$h_{ik}(t) = \sum_{r} {}_{r}h_{ik}(t)$$

$${}_{r}h_{ik}(t) = \frac{{}_{r}A_{ik}}{\omega_r[1-\xi_r^2]^{1/2}} \mathrm{Sin}[\omega_r(1-\xi_r^2)^{1/2}t]$$

Finite Element Model of Disc Head

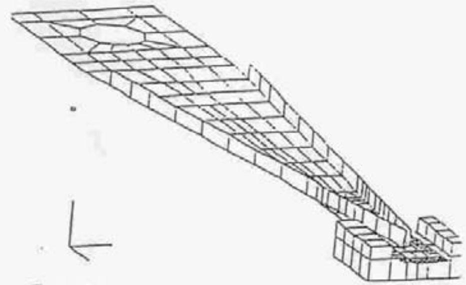


Fig. 8 Finite element model of read/write head suspension

In free head vibration test

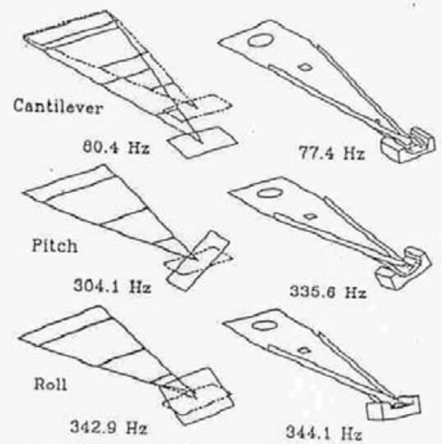
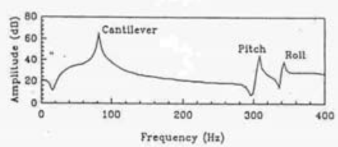
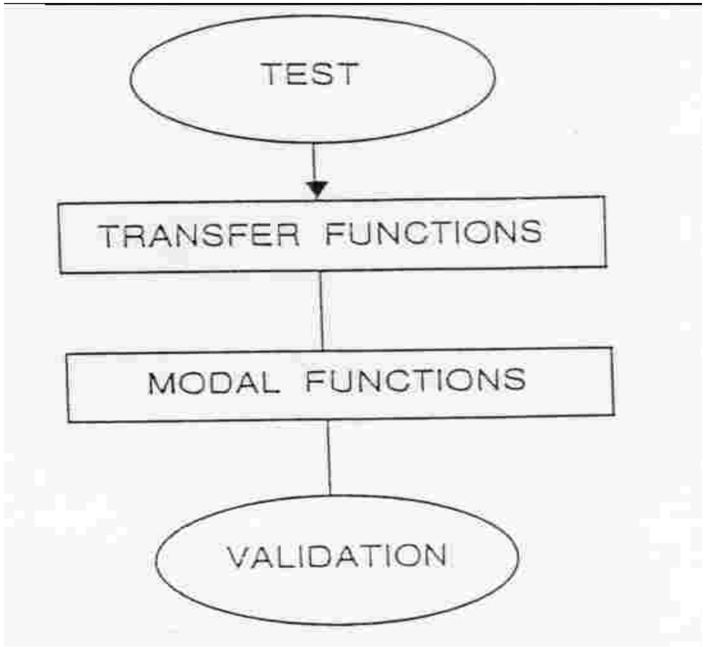


Fig. 7 Experimental (left) and numerical (right) mode shapes for free head vibration test (Compliance Modes)

Frequency Response Function (FRF) of head



Bode Plot



$$_rA_{ik} = \frac{_rA_{ij}\ \ _rA_{k1}}{_rA_{jj}}$$

$$H_{ik} = \frac{H_{ij}\ \ H_{kj}}{H_{jj}}$$

.: synthesize H_{ij} from H_{1j}, H_{2j} and H_{jj}

(response moved)

Signal Processing: Modal Analysis

$$
[H] = \begin{bmatrix} H_{11} & \cdots & H_{1k} & & \\ & & H_{2k} & & \\ H_{i1} & \cdots & H_{2k} & \cdots & H_{iN} \\ & & \vdots & & \\ & & H_{nk} & & H_{NN} \end{bmatrix} \leftarrow \quad \text{moving excitation}
$$

$$\uparrow$$

moving response

Moving excitation: Impact testing

Moving response: Shaker

Mathematical equivalence does not imply technology equivalence!

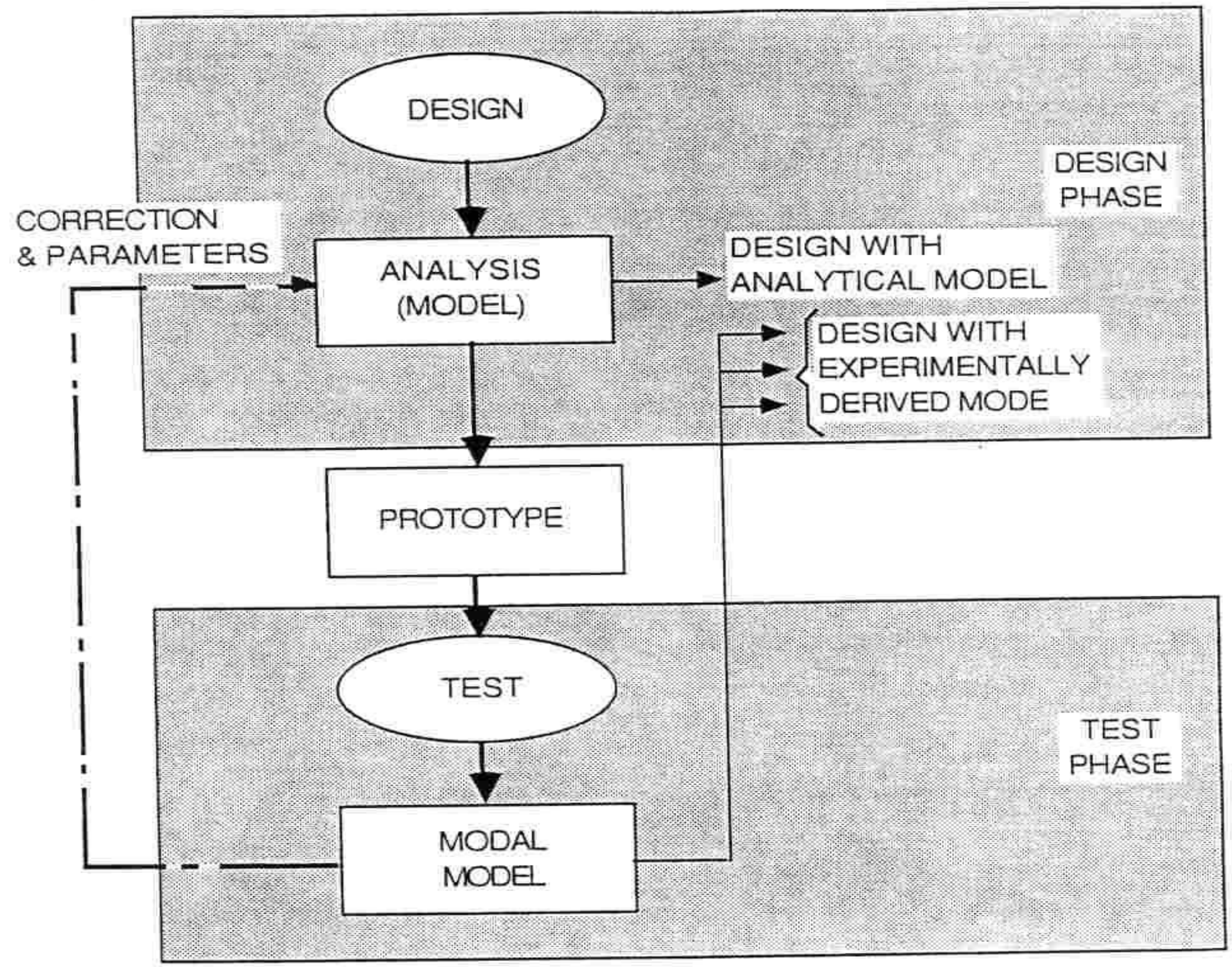
Modal Analysis applications

Model updating

Structural modifications

Diagnostics

Force evaluation via response



Signal Processing: Modal Analysis

STRUCTURAL MODIFICATIONS

$$\{[m]+[\Delta m]\}(\ddot{x})+\{[c]+[\Delta c]\}(\ddot{x})+\{[k]+[\Delta k]\}(x)=(f)$$

$$[\overline{\mathrm{M}}]=\begin{bmatrix}\diagdown & & \\ & \mathrm{I} & \\ & & \diagdown\end{bmatrix}+[\phi]^{r}[\Delta \mathrm{m}][\phi]$$

$$\left.\begin{matrix}\Delta \mathrm{m}\\ \Delta \mathrm{c}\\ \Delta \mathrm{k}\end{matrix}\right\}\ \text{modification matrices}$$

$$[\overline{\mathrm{K}}]=\begin{bmatrix}\diagdown & & \\ & \omega_r^2 & \\ & & \diagdown\end{bmatrix}+[\phi)^{T}[\Delta \mathrm{k}][\phi]$$

$$[\overline{\mathrm{C}}]=\begin{bmatrix}\diagdown & & \\ & 2\xi\omega & \\ & & \diagdown\end{bmatrix}+[\phi^{T}]\,[\Delta \mathrm{c}][\phi]$$

Find eigensolution for $[\overline{M}](\ddot{q})+[\overline{c}](\dot{q})+[\overline{K}](q)=[\phi]^{T}(f)$

Diagnostics

Change in Modal parameters

Frequencies (cracks etc)

Type (and changes) of deflection shapes

Force evaluation via response

$$[X]\exp(j\omega t) = [H](f)\exp(j\omega t)$$

$$(f) = [H]^{-1}(X)\exp(j\omega t)$$

Problem often ill conditioned

Appropriate numerical approaches needed